Introducing truth tables

Marianne: Hello, I'm Marianne Talbot and this is the first video in the series supplementing the Formal Logic podcasts.

Okay, introducing truth tables.

(Slide 2) This video supplements the second podcast in the Formal Logic series. You need to be ready to pause the video as we go through, if you want to try out the answers first for yourself, before seeing the answers that I give. Obviously, that's a sensible way to do it.

(Slide 3) We can formalise arguments in propositional logic only when their validity turns on truth functional connectives. That's a very important part of the limitations on propositional logic.

(Slide 4) A truth functional connective is a species of sentence connective. If you see the Venn diagrams I have got there in the bottom, sentence connectives, and then within that category, you have got truth functional connectives.

A sentence connective takes one or more sentences to make another sentence. (Slide 5) English has got lots of sentence connectives and here are a few of them.

'P and Q '- so the 'and' takes two sentences, 'P' and 'Q', to make a third sentence 'P and Q'. 'It is probably the case that P' takes one sentence to make another sentence so the sentence is 'P' and then we have, 'it's probably the case that P' is another sentence. All of those are sentence connectives, because each of them takes one or more sentences and uses them to make another sentence.

(Slide 6) A sentence connective is truth-functional when, and only when, the truth value of any sentence built from it, is a function *solely* of the truth values of its constituent sentences. So, we don't need to know the meaning of either of the constituent sentences to determine the truth value of the complex sentence. There are a lot of concepts in there that are quite difficult. You might want to linger over that slide and think about its meaning before you move on.

(Slide 7) Another way of putting that is that a sentence connective is truth-functional when, and only when, it has a complete truth table. Now that means that we can use truth tables to see whether we can complete them or not, to decide whether a sentence connective is truth-functional, whether it's a truth- functional connective or not.

(Slide 8) Truth tables enable us to decide whether a sentence connective is a truth-functional connective. As we will see in a later video, they also enable us to do other things, but this is the first thing that they allow us to do.

(Slide 9) Let's demonstrate this by drawing a truth table for 'P and Q' and seeing if we can complete it, thereby showing that 'P and Q' is a truth-functional connective.

Firstly, we have got to draw our empty truth table.

(Slide 10) Here we go, we draw a row of boxes and we have got the sentence connective in the main box, the longest box and then we have got the sentence letters of the connective in smaller boxes to the left of the main formula. If you look at the formula itself, that's 'P and Q', two sentence letters 'P' and 'Q' and so we have two smaller boxes to the left, one of which is 'P' and the other of which is 'Q'. If there were three sentence letters in the formula, we would have three boxes to the left,'P', 'Q' and 'R'.

(Slide 11) Now we need to add the appropriate number of rows and the rows are always according to the formula. If we have got one sentence letter, so if we have got ‘not P’ for example, we have only got one sentence letter, then we only have two rows because ‘P’ can be either true or false and ‘not P’ can be either true or false, so we only need two rows.

If we have got two sentence letters, as we have here, we need four rows because each of those sentence letters can have one of two truth values: true or false. So four, if we have got two sentence letters, we need four rows. If we have got three sentence letters, we need eight rows and so on.

We have got two sentence letters here so we need four rows and there’s our empty truth table. (Slide 12)

(Slide 13) The next thing we do is we add the truth values to the columns under the sentence letters. Now, we are still just drawing the truth table at the moment, so here we go.

(slide 14) If we look, we have got the ‘Q’ and in the first one to the left of the formula, we write true false, true false so they are alternating true false, true false. That is always the case. If we have got eight rows, we have got true false, true false, true false, true false, all the way down and that’s what we always have under the sentence letter of the one that’s immediately to the left of the formula.

(Slide 15) For the one that’s to the left of that one, we put true true, false false, and we do that again, all the way down. If there are 16 rows, we have got true true, false false, true true, false false, and so on. In this case, we have only got two sentence letters and so we have completed it in doing that. I hope you can see that here, we have got every single possible combination of the possible truth values for each of ‘P’ and

each of ‘Q’. They could both be true, they could both be false or they could each be one of each.

(Slide 16) Now, we have got to treat each row, or structure each row, on a truth table is called a ‘structure’, as a separate possible world. What we do, is we add truth values to the column under the sentence connective and we do that if we can. As you will see later on, it’s not always the case we can add a truth value to that column but with the ‘and’, I think you’ll find that we can.

(Slide 17) Looking at row one, forgetting the other rows for the moment, look at row one. This is the possible world in which ‘P’ is true and ‘Q’ is true. Can you tell me the truth value of ‘P and Q’ in that world? Is it true or is it false? In the world where ‘P’ is true and ‘Q’ is true, what is the truth value for ‘P and Q’?

Slide 18) I hope you’ll agree that it’s true. We can certainly put a truth value in there and now we have got to look at the next row.

(Slide 19) In the world where ‘P’ is true and ‘Q’ is false, what’s the truth value of ‘P and Q’? Can you tell me whether it’s true or whether it’s false? Decide that before you go to the next slide.

(Slide 20) I hope you agree that it’s false. In a world where ‘P’ is true and ‘Q’ is false, ‘P and Q’ has got to be false.

Now can you do rows three and four for yourself?

In the world where ‘P’ is false and ‘Q’ is true and in the world where ‘P’ is false and ‘Q’ is false, can you determine the truth value of ‘P and Q’? I hope you’ll agree that in the third world is also false (Slide 21) and in the fourth world it’s false as well. (Slide 22) We have got a complete truth table here. We can decide the truth value of ‘P and Q’ in every possible world and

that's without knowing anything about the *meaning* of 'P' or the *meaning* of 'Q'. We just need the sentence letters themselves and the truth values of those sentence letters.

(Slide 23) The sentence connective 'and' is truth-functional because it's got a complete truth table. Its truth value is a function solely of the truth values of its constituent sentences.

(Slide 24) The truth table for 'and' can be thought as the truth table definition of 'and' and that's because it gives us the truth value of 'and' in every logically possible world. To determine the truth value of a sentence constructed by 'and' in every logically possible world, simply, is to know the meaning of 'and'. It means you can determine the truth value and you can only determine the truth value of a sentence if you understand its meaning.

(Slide 25) Let's see if the sentence connective 'because' is truth- functional. We know that a sentence connective is truth-functional if we can draw a complete truth table. If we can draw a complete truth table for 'P because Q', then we know that 'P because Q' is truth-functional. If it doesn't have a complete truth table, then we know it's not truth-functional.

(Slide 26) We have got to start again by drawing our empty truth table. It's a binary sentence connective, it takes two sentence letters so actually, the empty truth table is exactly the same as that we drew for 'and'.

(Slide 27) Let's take the first row. This is the row in which 'P' is true and 'Q' is true. Can you tell me the truth value of 'P because Q' in the world where both 'P' and 'Q' are true? Okay, decide that before we move to the next slide.

(Slide 28) I hope you'll agree with me that we can't. Just because 'P' is true and 'Q' is true, that doesn't tell us the truth value of 'P because Q'. I mean, P might be the cause of Q in

this world or it might not be the cause of Q in this world. It depends what ‘P’ means and it depends what ‘Q’ means. It depends which states of affairs are referred to by ‘P’ and by ‘Q’. We have no idea of the truth value of ‘P because Q’ if all we know are the *truth values* of ‘P’ and ‘Q’.

(Slide 29) That tells us that the sentence connective ‘because’ is not truth-functional. We don’t have to bother with the other rows on that truth table because as soon as we have got a question mark, we know that the truth table isn’t complete and the sentence connective isn’t truth-functional.

To complete the truth table, we would need to know whether there is a causal relation between the events picked out by ‘P‘and ‘Q’. To know that we would need to know the *meanings* of ‘P’ and ‘Q’ and if their truth values alone are not enough to enable us to complete the truth table, then we know it’s not a truth functor. The sentence connective ‘because’ is not truth-functional. The sentence connective ‘and’ is truth-functional.

(Slide 30) There are five truth-functional sentence connectives in English. Here they are. Have a look at those because we will be dealing with those a lot.

(Slide 31) On the next slide you will find altogether on one truth table, the truth table definitions of each of these truth-functional connectives. You will actually find it in a much easier to read form in the hand-out booklet and that’s on pages 18-20.

(Slide 32) Here are all the truth table definitions. You will see that in the third row along, you have got the truth values of ‘not P’. You will see in the first row that ‘P’ is true, ‘not P’ is false. In the third row, if ‘P’ is false, ‘not P’ is true. The truth table definitions of all the other truth functors are given on that slide.

(Slide 33) That’s all you need to determine how to evaluate an argument with semantic sequent by means of truth tables, which we will do in a later video.