Testing semantic sequents with truth tables

Marianne: Hi. I'm Marianne Talbot and in this video we are going to look at testing semantic sequents with truth tables.

(Slide 2) This video supplements Session 3 of the Formal Logic podcasts and in it we shall be learning how to test semantic sequents by means of truth tables. Remember to pause the video if you want to try something for yourself.

You are going to need the truth table definitions of the truth functional connectives which you will find on pages 18 to 20, it says 10 there but it means 20, of the handout.

(Slide 4) As we saw earlier, truth tables are tabular representations of all the logically possibly situations generated by the combinations of truth values of the sentence letters within a formula. (Slide 5) You will remember that the argument claim made by a semantic sequence is, "There is no logically possible situation in which all the formulae to the left hand side of the turnstile are true and the formulae on the right hand side of the turnstile are false."

(Slide 6) Truth tables enable us to check whether it really is the case that there is no logically possible situation, no *structure* in which all the formulae to the left hand side of the sequent are true *and* the formula on the right hand side false.

(Slide 7) We will start by learning how to evaluate a very simple sequent, the sequent you can see on this slide.

(Slide 8) First we have got to draw the empty truth table, and the number of columns we need, the columns are the ones that run down, is the number of sentence letters in the sequent plus the number of formula in the sequent plus one for the sequent itself. So, how many columns do we need for the sequent that we've got? Work that out before moving on to the next slide.

(Slide 9) We need *six* columns to evaluate this sequent and there are the six columns there. I see on the third column it should be ‘P arrow Q’ and the ‘Q’ has slipped to the second line but that is ‘P arrow Q’.

(Slide 10) To determine the number of rows that we need, the ones running across, we need to look at the number of sentence letters that we have. You will remember from before, if there are two sentence letters we need four rows, because if there are two sentence letters each of them can have two truth values.

So, two sentence letters, four rows; three sentence letters, eight rows; four sentence letters, sixteen rows and so on. Have a look at the sequent that we are doing, how many rows do we need for that sequent? (Slide 11)

We need four rows in addition to the first one, the heading one for the truth table. Here is the empty truth table for this one.

(Slide 12) We have now got to complete the truth table according to the rules that you have got in your handbook.

First, let’s do the really easy columns. There are two columns with single sentence letters in them, and here all we do is copy the truth values from the key. (Slide 13) If you look at the ‘Q’ here and the ‘P’, both of those are formulas that are single sentence letters formulas.

If you look at the ‘P’ and look at the ‘P’ in the key on the left hand side, the ‘P’ on the left hand side is true-true-false-false and in the ‘P’ actually in the truth table itself I’ve got true-true-false-false. I’ve just copied it, nothing more complicated than that.

On the ‘Q’, if you look at the ‘Q’ in the key it’s got true-false-true-false and the ‘Q’ in the formula, true-false-true-false. Again, dead easy.

(Slide 14) Now we do the more complex formula. Usually we would start with the simplest complex formula but we've only got one in this sequence so that's the one we are going to do. The one we have is a conditional, so you need to look in your truth table definitions for the rule for the arrow, for the conditional sign.

(Slide 15) Under the arrow we inset the truth values given to us by that rule. If you look under the 'P arrow Q' in the first row, the row in which 'P' is true and 'Q' is true, the truth value for 'P arrow Q' is true.

In the second row, ignore all the other rows as you are doing one row. We are looking at row 2 now, the world in which 'P' is true and 'Q' is false, and 'P arrow Q' in that world is false. 'If P then Q' is false, if 'P' is true and 'Q' is false. The rule will tell you that in the world where 'P' is false and 'Q' is true, 'if P then Q' is true and where they are both false, 'if P then Q' is true again.

Don't worry about trying to understand these truth values at the moment; all you are trying to do at the moment is to apply the rules. There are complications with the truth table definition of the arrow and you will find that explained on the last podcast in the Formal Logic series. It's one of the questions people ask me when the question time comes, but don't worry about that at the moment. All we are doing at the moment is learning how to apply the rules.

(Slide 16) We have now put in all the truth values and the time has come to actually evaluate the sequent. (Slide 17) We know that a sequent is incorrect if and only if there is a logically possible situation in which all the formulae to the left hand side (LHS) of the sequent are true and the formulae to the right hand side (RHS) of the sequent are false. That is the only situation in which the sequent is incorrect.

(Slide 18) Let's look across the rows to see if there is a situation in which there are only Ts for true to the left of the turnstile.

(Slide 19) Row 1, I think you will agree, is the only row in which all the formulae to the left of the sequent are all true. Notice that I have put those both in black. If you look at the other three rows, in each of them you've got a 'true' and a 'false', so it's only Row 1 where you have got two 'trues'.

(Slide 20) So it's only Row 1 that is the possible world, or as you know they are called 'structures' in a truth table, Structure 1 is the only one where both the formulae to the left of the sequent are both true.

So that's the only one we look at when we need to know whether the formula on the right hand side is false. If the formula on the right hand side of the sequent in Row 1 is false, then the sequent is incorrect.

(Slide 21) But the formula to the right of the sequent in Row 1 is *not* false, it's true. So we can put a tick under the sequent there: there is *no possible situation* in which the formulae to the left hand side of that sequent are all true and the formula to the right hand side is false. There is no possible situation in which that is the case, therefore the sequent is correct.

You might like to run through that again before we go on, because we are about to go on to a more complicated sequent. You might want to look at it again for the very simple sequent before we go on to the complicated one.

(Slide 22) Now, let's evaluate the sequent with which we started. There's the argument, do you remember what it was about? (Slide 23) Here is the interpretation, so you can go back to that sequent, you can look at that interpretation and you can remind yourself what the argument was about.

Actually in evaluating the argument and using the truth tables to see whether the argument claim is true or not, we don't have to bother with the interpretation, so if I were you I wouldn't bother to interpret it all unless you want to remind yourself what the argument was.

(Slide 24) To test this sequent we first draw the empty truth table, as always. Here we've got *four* sentence letters, so we need sixteen rows in addition to the first row. (Slide 25) I would just like to say, this is why we use tableau, because actually doing truth tables when they've got more than two, or at most three, sentence letters, becomes a real pain, it gets quite complicated. (Slide 26) As you can see, it gets quite complicated, that's how long our truth table is.

Notice that under the sentence letter to the immediate left of the sequent I've got true-false-true-false-true-false-true-false all the way down, just alternating true-false. To the one immediately to the left of that I've got true-true-false-false-true-true-false-false all the way down. To the one immediately to the left of that, I've got four Ts then four Fs then four Ts then four Fs. To the one to the left of that, I've got eight Ts then eight Fs and if it were even longer God forbid, then it would go on like that.

So make sure you know how to draw an empty truth table before you do any testing of any complex sequent because you'll get into a real mess if you don't know how to draw the empty truth table.

(Slide 27) Now we've got to start completing the truth table itself, and first we will do the most complex formula which is the one on the right hand side of the sequent. (Slide 28) That's the one I mean, it's the most complex one because it has got two truth functional connectives: it has got the negation sign and the conditional, and so it's got two truth functors where all the other formulae in the sequent have only got one.

So that's the one we are going to do first, and the first one we do we start completing and we do this in lower case letters.

(Slide 29) The truth value of the negated antecedent of that conditional, because this is dead easy, it's a P that is negated and all we need to do is reverse whichever truth value P has in the key.

(Slide 30) This is what we're doing. If you look, we are just doing the 'not P'. We are ignoring the rest of the conditional at this point. Can you go through and put down under that 'P' in lower case letters whatever truth value 'not P' has when 'P' is whatever truth value is in the key?

I hope this is what you have. (Slide 31) There are eight trues and then eight falses, and under 'not Ps' you've got eight falses and then eight trues. Dead simple, very easy and again it shows you why truth tables are actually quite boring but hugely useful.

(Slide 32) The next thing we are going to do is to complete, again in lower case letters, the truth value for the 'S' that's that consequent of the conditional on the right hand side of the sequent. This will help us get the truth value of the whole conditional right.

(Slide 33) If you look at the S there, I've got lower case letters again and I've just copied what is in the key for 'S' under the 'S' on the truth table. Check out that you see exactly what I mean by that before we move on. So truth-false-truth-false all the way down.

(Slide 34) Now we can complete the truth value of the conditional itself, and this time we do it in upper case letters because actually this is the truth value we are interested inWe only completed the lower case ones to make it easier to complete overall the truth functor with the largest scope.

.(Slide 35) If you look at the rule for arrow, you will see that when you've got a 'false' for the antecedent of the conditional and a 'true' for the consequent, the whole conditional is true. On Row 2 we've got false-false so the whole condition is true, and if I go down here, if we look on the row where you see a 'false' for the first time under the conditional, that's because you've got a true antecedent and a false consequent. That makes the whole conditional false.

Make sure you understand where all those truth values are coming from before you move to the next slide,

(Slide 36) Now let's do the first formula, the one to the left-most of the formulae on the left hand side. This is a *disjunction* and you will see on the rules that you are following in your handbook that that has the following truth table. What is really important here is you use the right part of the key. It's very easy to get confused with the 'P's, 'Q's, 'R's and 'S's when you've got so many of them.

(Slide 37) If you look down, I have filled these in with black letters, so I've highlighted in black the truth values under the 'P' and the 'Q' in the key, and the truth value of 'P or Q' are these truth values underneath the 'P or Q'.

So again, looking at Row 1, in the world where 'P' is true and 'Q' is true, 'P or Q' is also true. Let's go down until we find a false, and that's the world where 'P' is false and 'Q' is false, 'P or Q' is false in that world.

Again, make sure you understand where all those trues and falses are coming from before you move on to the next slide.

(Sl;ide 38) The next slide we are going to do is the 'Q arrow R'. No, okay, I'm going to ask you to do the two final formulae for yourself and remind you again, do make sure that you are looking at the right sentence letters in the key as you work out the truth values that should be in the truth tables.

Complete those final two columns for yourself before you move on.

(Slide 39) Here is the ‘if Q then R’. I have highlighted in black the parts of the key that we need, and I have highlighted in black the truth values of ‘Q arrow R’ for each of those keys. Make sure you know where all those trues and falses have come from before you move on to the next one.

(Slide 40) The final one, ‘R arrow S’, again I have highlighted in black the parts of the key that we need and the truth values that we put in according to the rules. Again, make sure that you know where all those ‘true’s and ‘false’s are coming from before you move on to the next one.

This is the last one actually; the next one we are going to do is the sequent itself. Make sure you know where all the trues and falses come from before you move on.

(Slide 41) Now we can check the sequent itself. We need to know whether there is a situation, a *structure* or a *row* in which the formulae to the left hand side are all true and the formula to the right hand side false. If there is such a structure, then we know that the argument claim is incorrect. (Slide 42)

Is there such a structure? (Slide 43) Well, there are five situations in which the formulae to the left hand side are all true, but in each of these situations the formula to the right hand side is true as well. So there isn’t a single situation in all 16 rows of that truth table where the formulae to the left hand side of the sequent are true and the formula to the right hand side is false.

There isn’t a single logically possible situation where that is the case, (Slide 44) so we can see that we can put a tick in each of those rows showing that in each of those rows the sequence is correct. (Slide 45) The other rows don't matter so much because in none of them is it the case that all the formulae to the left hand side are true.

As you look down you will see that all of them have got a mixture of 'true's and 'false's, so in all the rows where the formulae to the left hand side are all true, the formula to the right hand side is also true. So, a tick under the sequent in each of those.

(Slide 46) So, there is no logically possible situation in which all the formulae to the left hand side are false and that on the right hand side true. This tells us that the sequent is correct and the argument is valid.

(Slide 47) So, this is the argument. If you remember, that's the argument we started with. A very complex argument, you don't even know how to go about evaluating it, but you now know that it's valid. (Slide 48) You also know how to test a semantic sequent by means of truth tables. So, well done.