Formalising arguments

Marianne: Hi, I'm Marianne Talbot and this is the first of the videos that supplements the podcasts on formal logic. (Slide 1) This particular video supplements Session 2 of the formal logic podcasts, and we're going to look at formalising arguments.

You're going to find it very useful to have the handout booklet for the lecture series ready as you do this. If you'd like to try what I'm going to do as we go along – and I think you should – be ready to pause the video so you can answer for yourself before revealing my answer.

(Slide 3) To formalise an argument, we've got to translate the English of the argument into the language of propositional logic. (Slide 5) We'll start with this very simple argument; have a look at the simple argument. (Slide 6) Our next step or our first step is to identify the simple sentences that make up the argument. Our simple sentence is one that doesn't have sentences as parts, so all the parts of the sentence are sub-sentential.

(Slide 7) Here's the argument again; can you identify the simple sentences in this argument? (Slide 8) One simple sentence is: 'It's Friday'. I've outlined those in red so you can see that, and there's only one other sentence: 'Marianne is wearing jeans'. (Slide 9) Again, we've put that one in red for you.

(Slide 10) There are only two simple sentences that make up the simple argument that we're doing: 'It is Friday', and 'Marianne is wearing jeans'. (Slide 11) Our next step is to give an interpretation for the argument. (Slide 12) To do this, we

give each sentence a sentence letter. A sentence letter is a capital letter taken from ‘P’, ‘Q’, ‘R’, ‘S’ part of the alphabet. That's convention, but it's quite an important convention because, as we’re learning at the moment, it would be nice if everyone formalised arguments in the same way – much easier to check them against each other. Here's our interpretation for the simple argument that we're doing.

We then use the interpretation to replace the sentences in the argument with the sentence letters of the interpretation. (Slide 13) You should do that for yourself before we turn to the next slide. We replace each instance of ‘It is Friday’ with ‘P’, and you'll see that I've done that here, (Slide 14) and (Slide 15) we replace each instance of ‘Marianne is wearing jeans’ with Q, and I've done this here again.

This is our argument now. (Slide 16) As you can see, we've stripped the content from it; nothing is left except the logical words ‘if’ and ‘then’. (Slide 17) Our next step is to choose the appropriate truth-functional connective for the complex sentence of the argument.

(Slide 18) In this argument we have only one complex sentence, the ‘if P then Q’, so which truth-functional connective do you think we will use for this premise? (Slide 19) If you want to check the list of truth-functional connectives, you'll find them on page 9 of the handout booklet or on slide 49 of Session 2.

The truth-functional connective we should use for ‘if P then Q’ is the ‘if then’ connective; not rocket science, is it? That’s the conditional or the arrow. (Slide 20) There we have the argument with the arrow replacing the ‘if then’.

(Slide 21) At this point we should add two brackets for every *binary* truth-functional connective. A binary truth-functional connective is one that takes two sentence letters, like ‘if P then

Q'; a *unary* one is one that takes only one, like 'it is not the case that P'. (Slide 22) Our argument is very simple; we have only got one truth-functional connective and that's the binary one arrow, so we need only add brackets around the formula that is premise one, as I've done here.

(Slide 23) The next step of the formalisation is to provide the sequent. Here are the symbols for the sequents. There's the semantic sequent, which is the turnstile with two crossbars, and the syntactic sequent, which is the same turnstile but with only one crossbar. In this video we're doing only the semantic sequent.

(Slide 24) The sequents formalise the argument claim itself, so whenever anyone puts forward an argument, they're actually performing an *action*; they're *making a claim*. The claim that they're making is that the conclusion follows from the premises, or we could also think of it as: 'there is no logically possible situation in which the premises are all true and the conclusion false', or 'the argument is valid', or 'the premises entail the conclusion'. There are lots of different ways in which we can express the argument claim, but basically the sequent encapsulates the claim that *is* the argument.

(Slide 25) Sequents are always set out along the page, not down it, as I've done here; I've set it out along the page. This is a semantic sequent; as you can see, the turnstile has a double crossbar. (Slide 26) We read that as: there is no structure – and you'll see what I mean by 'structure' when we do the truth tables in a later video – there is no structure in which the formulae on the left-hand side of the sequent are all true and the formula on right-hand side of the sequent is false. That's the *meaning* of the semantic turnstile.

(Slide 27) If we want to formalise the sequent as a *syntactic* sequent, we write that, and that's a syntactic sequent because

it had a single crossbar. We read that as the tableau generated from the set, curly brackets, *counterexample set.* Notice that the two premises are there and then there's the negation of the conclusion and no turnstile, so that's the set that has to be closed if that argument claim is correct. The tableau generated from the counterexample set is closed.

(Slide 28) The difference between the semantic sequent and the syntactic sequent is that the latter represents a deeper level of formalisation. We don't need to refer to truth or possibility in understanding it; instead, we formalise both those notions.

(Slide 29) Now let's do a complex argument. Here's the complex argument that we're going to do, and I think you'll agree that if you look at that – very difficult see how to go about evaluating that, a lot of noise in that argument. (Slide 30) I've analysed the argument for you, in accordance with the steps in Session 2, so this is the argument as analysed. We *always* analyse an argument before formalising it; it's a mug's game to try and formalise an argument without having analysed it.

(Slide 31) First, we identify the simple sentences of this argument, just as we did with the very simple argument before. (Slide 32) The simple sentences, just to remind you, are those sentences, all the parts of which are sub-sentential. 'The cat is on the mat and the dog is at the door' is complex, whereas 'The cat is on the mat' is simple, because all its parts – 'the cat', 'is', 'on', 'the mat' – are all simple, they're not sentences themselves. (Slide 33) So, can you identify the simple sentences of the argument before we move on to the next slide?

(Slide 34) The first simple sentence is: 'She didn't want you to tickle her', and we've identified those in red. There are two

instances of those and you'll notice that one of them is the negation of the other. The first one (in premise one) is ‘She didn't want you to tickle her’, but the one in the conclusion is ‘If she did want you tickle her’. This is the *same* sentence, but one is the negation of the other. We don't need to worry about the negation, because we can capture that by the truth function itself, so we don't need to worry about that in the interpretation.

(Slide 35) The next simple sentence is: ‘She didn't realise you were going to tickle her’, and I've highlighted both of those in red, so again you can see that there are two instances in our argument. (Slide 36) The third simple sentence: ‘You were going to tickle her in the wrong way’, again two instances in the argument; I've highlighted them in red. (Slide 37) Finally, there's the simple sentence: ‘You deserve to get scratched’; again, two instances, one of them in the conclusion, one of them in premise three.

(Slide 38) Here are our four constituent sentences for that really quite complex argument. (Slide 39) So, our next step is to provide the sentences with sentence letters, and in doing that we provide an *interpretation* for the argument. The point of providing an interpretation is so that when we've evaluated the argument, we can reinterpret it into the English and get back what we actually want to know, which is the content of the argument.

(Slide 40) Here's the interpretation. I've used the sentence letters ‘P’, ‘Q’, ‘R’, ‘S’ again. If you need to go on, there's ‘P’, ‘Q’, ‘R’, ‘S’, ‘T’, ‘U’, etc., although we try and keep arguments simple, so I hope in learning you won't do more than four or five. (Slide 41) The next step is to substitute the sentence letters of the sentences for the sentences; in doing that, you formalise the argument, or at least partly formalise it.

(Slide 42) The first simple sentence is: ‘She didn't want you to tickle her’. We're using ‘P’ for that sentence and here I've put ‘P’. Remember that we negated that very sentence in the conclusion, so ‘P’ in the first premise but ‘not P’ in the conclusion. Be very careful with that, because that’s a time when it's very easy to make a mistake to forget to negate one of the sentence letters.

(Slide 43) The next simple sentence: ‘She didn't realise you were going to tickle her’, we use ‘Q’ for that, and you'll see that I've highlighted the ‘Q’ that I've replaced for that sentence in each case.

(Slide 44) Third simple sentence: ‘You were going to tickle her in the wrong way’. We used ‘R’ for that, two instances of ‘R’, and there they are. (Slide 45) Finally, there's the simple sentence: ‘You deserve to get scratched’. For that, we used the sentence letter ‘S’, one in the third premise and one in the conclusion.

(Slide 46) Here's our partly formalised argument. Again, that was quite easy, wasn't it? I hope you think so. The next thing we've got to do is to formalise the truth-functional connectives and add brackets. (Slide 47) All we've got left at the moment is the logical words, and what we've got to do is replace all those logical words with truth-functional connectives of propositional logic. You should try that on your own before we move to the next slide.

(Slide 48) Here are the truth-functional connectives that I've chosen; I hope they're the ones that you've chosen. Quite simple this argument; I wish I could promise you that all arguments are that simple. It's not the case, but this one you're learning, so I hope you’ve got the same truth-functional connectives as I’ve got there.

(Slide 49) An important thing to note is that *without the brackets* the last formula is ambiguous. If we take the brackets out, that's the last formula, ‘not P arrow S’, and that could be (not-P then S), or the negation *then* (P then S). The first one reads: ‘if it's not P, then S’ and the second one reads: ‘it is not the case that if P then S’.

I hope you'll agree that those are two very different sentences. The brackets, therefore, are very important, because they disambiguate that sentence; they make it clear which meaning is intended.

(Slide 50) The next thing we do is provide the sequent, the argument claim itself. (Slide 51) We do this by listing all the premises, separated by commas on the left-hand side of the sequent, and then we add the conclusion on the right-hand side of the sequent. Remember it goes along the page; all sequents go along the page, not down the page.

Here's the semantic sequent for our argument. (Slide 52) Remember that this is read as: there is no logically possible situation – or I might have put ‘no structure’, which is a logically possible situation in a truth table – there's no logically possible situation in which the formulae on the left-hand side of the sequent are all true and the formula on the right-hand of the sequent is false. That's the meaning of the semantic turnstile that we've got there.

(Slide 53) Here's the syntactic sequent. As you can see, this is just one crossbar on the turnstile. That's read as the set, and then we've got the curly brackets and then the counterexample set within the curly brackets. That set is closed; that's what the syntactic sequent means. As you can see, we're not talking about truth or possibility anymore.

(Slide 54) So, we started with this argument, a very complex argument, very difficult to know how to even go about evaluating it, and we ended with that very simple sequent. (Slide 55) Very important to remember that there are many pitfalls you're going to meet as you formalise English arguments as sequents of the propositional calculus.

The ones we've done here have been very simple; they're not all going to be that simple and you will meet these pitfalls. The pitfalls themselves are discussed quite briefly in the podcast for Section 2, but they're discussed much more fully in the course textbook itself, Critical Reasoning: A Romp Through the Foothills of Logic. I really do recommend that you read those very carefully before you go on there.

That's the end of this podcast. I hope it's been helpful for you; very good luck with formalising arguments.