A romp through the foothills of logic – Session 4

Lecturer: (Slide 2) In the last session we looked at the evaluation of semantic sequents using truth tables. What we haven't done at the moment is looked at evaluating syntactic sequents.

Now, those of you who think that we could probably do syntactic sequents by the means of truth tables, you are absolutely right, but really, you shouldn't.

Syntactic sequents are a deeper level of formalisation and then it's not defined in terms of truth and falsehood, and so actually the truth tables shouldn't be used to evaluate syntactic sequents.

I'm going to show you another way of evaluating syntactic sequents, and that's by *tableau*, or sometimes people call them 'truth trees'.

(Slide 3) The difference between semantic – can anyone tell me the difference to look at [the symbols]? How do you know whether a sequent is a syntactic or a semantic sequent?

Male: Because there's a line.

Lecturer: That's right. The turnstile would be a single one if it's a syntactic sequent and a double one if it's a – ooh, thanks for putting up the flipchart – That's a syntactic sequent (the single crossbar) and that's a semantic sequent (the double crossbar). That's how you know the difference.

Male: Isn't it the other way around?

Lecturer: What did I say?

Male: You said...

Lecturer: Semantic, syntactic.

Male: No, you said it the other way around.

Lecturer: Oh, right. Let me start again. Semantic (double crossbar), syntactic (single crossbar).

That's (the semantic sequent) defined in terms of truth and possibility. (Slide 4) Whenever we see that, we say that the claim being made by this sequent is that *there is no possible structure, or row on a truth table, in which all the formulae to the left-hand side of this sign are true and the formulae to the right-hand side are false*. That's how we define that.

(Slide 5) Whereas this one (the syntactic sequent) is defined in terms of the set consisting of those formulae (those on the LHS), and the negation of those formulae (those on the RHS) is closed: it generates a closed tableau. I'm going to show you what that means today.

So semantic sequents, with double turnstiles, are defined in terms of truth and falsehood, and we test them by truth tables.

The claim is there's no possible structure in which all the formulae to the left are true and the formulae to the right are false.

If there is a sequent in which all the formulae to the left are true and the formulae to the right are false, you will find it with a truth table, because you will look down the sequent, you will look down the rows, and if there's anything with all the Ts to the left and an F to the right, that's a counterexample. You put a cross in under the sequent there.

Syntactic sequents are not defined like that, because the argument *claim* made by a syntactic sequent is that the set consisting of those formulae (those on the LHS) and the negation of those formulae (those on the RHS) generate a 'closed tableau'.

So we're not mentioning truth and falsehood. How could that claim be tested in a truth table? So that's really why you shouldn't test syntactic sequents with a truth table.

Believe me, tableaux are much easier than truth tables, but you're not going to think that for at least an hour. (Laughter) I hope you might think it after an hour.

The semantic sequent 'if P then Q', 'P', the semantic sequent, 'Q' is interpreted, there is no logically possible – we've done all this – there's no logically possible situation in which all the formulae on the left-hand side of the sequent are true and those on the right-hand side false. You saw yesterday how we use truth tables to test that.

The little curly brackets there make a set. This sequent is interpreted 'P arrow Q', 'P' and 'not-Q' – see, we've put a negation sign in front of Q – is closed.

So this set {'if P then Q', 'P' then 'not-Q'} (Laughter) So the same is true here. 'P arrow Q', 'P' 'not-Q', is a counterexample set for the sequent above.

A counterexample set you get by just putting all the formulae on the left-hand side, just as they are, and then adding the *negation* of whatever is on the right-hand side.

So it's by generating a tableau, or a truth tree, from the counterexample that we see whether or not the sequent is correct, because what you will see is what it means for a tableau to be closed. The tableau generated by the counterexample set is closed, and that's what you're going to learn.

(Slide 6) Tableaux are always generated from the counterexample set of the sequent being evaluated.

(Slide 7) The counterexample set of an argument consists in the premises of the argument plus the negation of its conclusion.

If you're thinking, "Why is she talking about arguments?" you're quite right, because I really should be talking about sequents, shouldn't I, because we've got rid of the argument in favour of a sequent?

It should be the counterexample set of the sequent consists in the formulae to the left-hand side, plus the negation of the formula to the right-hand side.

(Slide So if your formalised argument is that (**(P → Q), P ├ Q**), then the counterexample is <u>that:</u>

(P → Q)

P

Q

Notice I've put it down the page here. Unless you put the curly brackets on, to show it's a set, the counterexample set you should be putting *down* the page, because that's the set from which you generate the sequent, and you will see that we need to be *down* the page for that.

So the sequent goes *across* the page, the counterexample set goes *down* the page.

(Slide 9) In drawing the tableau, we take each formula of the counterexample set, and we represent its truth conditions according to the rules of propositional logic.

As we generate the tableau, we see the combination of these truth conditions, and that enables us to decide whether or not the sentences of the counterexample set are contradictory or not, or consistent or not.

A set of sentences is consistent if, and only if, and remember that's ('iff') not a mis-spelling, there is a logically possible situation in which all are true together.

This will make more sense when I show you a tableau, so we might come back onto it at that point.

(Slide 10) So if an argument is *valid*, its counterexample set will be *consistent*. [ooops I am making a mistake here as one of the participants points out! MT] Can you see that?

Male: Inconsistent.

Lecturer: *Inconsistent*, you're absolutely right. So an argument is valid – who can give me the definition of validity?

Male: There's no …

Lecturer: Kirsty is trying to avoid my eye. (Laughter) Is everyone else trying to avoid my eye? What were you going to say?

Male: There’s no possible occasion when the left side of the formula can be true and the right side isn’t true as well?

Lecturer: You're getting there. That’s not the definition of validity. That’s the definition of a semantic sequent. So an argument is valid if, and only if, there is no...?

Male: Situation where if the premises are true the conclusion is false?

Lecturer: Say it again.

Male: There’s no situation in which if the premises are true the conclusion is false?

Lecturer: Okay, except you should have put ‘logically possible’ in there. An argument is valid if, and only if, there is no logically possible situation in which its premises are true and its conclusion false.

Of course, in making a counterexample set, what you're doing is you're putting the set of the premises and the negation of the conclusion.

Now, that set ought to be *inconsistent* if the argument is valid. Because if there’s *no* situation where the premises are true and the conclusion is false, i.e. the negation of the conclusion is true, then that set ought to be inconsistent, i.e. such that there is no logically possible situation in which all those sentences are true together. So if an argument is *valid*, its counterexample set will be *inconsistent*.

I will warn you now, because I've taught this for many, many, many years, and it always happens. I tell people this and they forget. Because you think validity is nice, consistency is nice, therefore valid and consistent go together, but that's not the case.

If an argument is valid, its counterexample set will be *inconsistent,* for the reasons I've just told you.

Similarly, if an argument is invalid, then its counterexample set will be *consisten*t, because for a set of sentences to be consistent is for there to be a logically possible situation in which they are all true together.

Well, if there's a logically possible situation in which the sentences of the counterexample set are all true together, it shows the argument must have been invalid.

Male: I struggle with that one a bit, I must admit. It doesn't seem blindingly obvious, like the first one.

Lecturer: It is blindingly obvious. It is as blindingly obvious as the first one.

Because the counterexample set is the set consisting in the premises plus the negation of the conclusion. If an argument is valid *only* if there's no possible situation in which the premises are all true and the conclusion false, if that set is consistent, i.e. such that they could all be true together, so the negation of the conclusion is true at the same time as all the premises, then that is a possible situation in which the premises are all true and the conclusion is false, isn't it?

Male: Okay.

Lecturer: If the counterexample set is consistent, the argument is invalid. The tableau tells us whether the counterexample set is consistent or not.

The tableau generated by a counterexample set is closed if all its branches are closed. You don't know what a branch is yet.

(Slide 11) A branch of a tableau is closed if, and only if it contains a simple formula, for example 'P' and its negation, 'not-P', or if it contains 'Q' and 'not-Q', or 'R' and 'not-R', and so on.

Because, given that we're assuming bivalence – do you remember I said that in classical logic, which is what you're being taught here, it's always assumed that a sentence is either true or false; there's no third truth value, and there's no truth value gap. If a sentence isn't true, it's false, and if it isn't false, it's true, and so on – so if there's a formula and its negation within a tableau, we can close that as being contradictory. We have failed to draw a situation where all the sentences are consistent with each other. Here's an obvious situation where they're not consistent with each other.

Again, don't forget I'm going to show you all this in a minute, and so this will start to make sense.

Male: Is that the only case that you can close?

Lecturer: Yes. You can only close a tableau if there’s a formula and its negation in it. I will add to that later on, but not just now.

There is another situation, but it is in effect the same situation but we come at it in another way, and you really don’t need to know that now, so we won’t. We will forget that.

Male: That’s fine.

Lecturer: (Slide 12) Note that ‘P’ and ‘not-P’ give us a structural way of determining inconsistency.

So consistency is defined in terms of truth again, and I was saying that we’ve got rid of truth. Well, I'm explaining it to you in terms of consistency and truth, because otherwise you won’t understand it, but actually it’s this structural inconsistency.

Computers can recognise that. If you have the symbol ‘P’ and the symbol ‘not-P’, the computer knows that that’s not possible.

So actually we don’t need to talk about truth or falsehood at all, and if you had time to really get into the language of propositional logic you would find you wouldn’t have to mention truth or falsehood.

(Slide 13) So, can you create counterexamples for the following sequents? What do I do with this one in order to create the counterexample? Put your hand up when you’ve got the answer. Right up so I can see you.?

Female: ‘Not-P’ on the left-hand side?

Lecturer: That’s right. You take out the sequent and you put ‘not-P’, and that’s the counterexample set. Of course you would run it down the page instead of across.

What’s the next one? Are you putting your hand up Sean or are you scratching your ear?

Mary?

Sean: I put my hand up as well.

Lecturer: Well, you were scratching your ear, so you're too late now. (Laughter) Mary, what would you put here?

Mary: ‘R’ on the right-hand side?

Lecturer: You get rid of the sequent, and actually you put ‘not-not-R’, but that is actually ‘R’, so you could put ‘R’ if you like.

If you were an undergraduate I would stop you doing that. The reason I would stop you doing that is that the computer programme that I would send you to, to practice on, would only recognise ‘not-not-R’ there, because it’s very pedantic. You’ve got to go through all the rules. It’s only a computer programme.

Can you see that you would negate the conclusion, which is already a negation? So it’s ‘not-R’. You put ‘not-not-R’. Of course, because of bivalence, ‘not-not-R’ is in fact ‘R’.

What about the last one? Steven?

Steven: The same as the first one, ‘not-P’.

Lecturer: Good. You take away the sequent, put ‘not-P’, and run it down the page.

Good. You’ve all got the idea.

Male: So when you put ‘not not R’, is that really just pedantry, or are there situations in which ‘not not R’ actually crawls out of the woodwork as being something a bit different from R?

Lecturer: Not in classical logic. In classical logic we assume bivalence, which is that if it isn’t true, it’s false, and if it isn’t false, it’s true. There’s no third truth value, no third possibility.

Male: So that pedantry is used because later on, when you get into fuzzy logic or something, [inaudible] …

Lecturer: Well, fuzzy logic is one of them, and also intuitionism. So any logic that doesn’t have the rule ‘not-not-P’ is ‘P’, you certainly wouldn’t be able to do that.

Male: When you set your students up with this pedantry, what you're doing is preparing them for a world in which bivalence isn’t

	assumed? You're setting them up with a language that doesn’t assume ...?
Lecturer:	Well, no. What I'm setting them up for is the computer programme that they can practice on, which is pedantic.
Male:	It is just …
Lecturer:	Yes, because there is a programme. You would be able to use it, if you're on the Oxford system. Actually, you can get into it from Google.
Male:	If it’s ‘Tableau 3’, then yes, I've got it.
Lecturer:	Yes. You’ve got into it from Google, haven’t you? So there’s something called ‘Tableau 3’, let me tell you now, which is the University of Oxford’s teaching tool for its undergraduates, and you can go to it. You’ve got to download an applet or something, haven’t you?
Male:	I didn’t, no.
Lecturer:	I had to, because it’s just gone from my computer.
Male:	I'm not using it fully, so maybe there is...?

Lecturer: If you're going to do the tableau, you do have to use that (you don't have to but you can).

Male: It looked like it was fully functional.

Lecturer: But it will give you not only tutorials on everything I've told you over this weekend, but it will also give you sequents, and it will allow you to apply the rules.

It's very fiddly, and it's very pedantic, but it's extremely useful, because you can test sequents. It won't let you...

It tells you that you're mind-numbingly stupid if you get one of the rules wrong. Undergraduates love that. You might find it a bit rude. (Laughter)

Male: Just out of slight curiosity, are we going to develop 'not-not-not-P'?

Lecturer: Well, what do you think 'not-not-not-P' means?

Male: Well, that's 'not-P'.

Lecturer: Yes.

Male: But in our calculations that we're going to do later, are we going to build up sequences of nots?

Lecturer: Not like that, no, because it would be a bit pointless, wouldn't it? It's so obvious.

Male: I thought the 'not-not…' was

Lecturer: We are going to build up sequents, but not like that.

Male: There will be a rule that only two nots would be … if you do any calculation, you get rid of this.

Lecturer: Well, it would be the same.

Male: You would never have a third one there.

Lecturer: Yes. (Slide 14) When we generate a tableau, what we're doing is testing the set of sentences consisting of the premises, and the negation of the conclusion, to see if there's a logically possible situation in which they're all true together. That's what we're doing.

(Slides 15 & 16) Oh, dear! Yawn, yawn, yawn, yawn. (Laughter)

(Slide 17) So consistency means invalidity and inconsistency means validity. That's the counterintuitive thing.

(Slide 18) Once we've created the counterexample set our next task is to generate the tableau. We do this following the tableau rules of propositional logic. (Slide 19) These rules are generated by the truth table definitions of the truth-functors.

If you remember, when we got the truth table definitions I said that this was a definition because it captures all the conditions, the truth conditions, all the logically possible conditions under which this is true and it's false. Well, the tableau rules do the same thing.

(Slide 20) Here we've got 'P and Q', and here's the truth table set up as always, true false, true false, true true, false false. Where 'P' is true and 'Q' is true, 'P and Q' are true. Where 'P' is true and 'Q' is false, 'P and Q' is false.

Do you remember? We did this yesterday.

Where 'P' is false and 'Q' is true, 'P and Q' are false again. Where they're both false, it's false again.

That's a truth table definition of 'and' (on the LHS). This (on the RHS) is the tableau rule for 'and'.

The tableau rule tells us that this is true in one world where both 'P' and 'Q' are true. So you've got a branch [I should have said 'stick' here - MT] on the tableau, and 'P' and 'Q' on the same stick. So it's true when these two are true and false otherwise.

Let's just do the next one and give you a feel for it.

(Slide 21) This is the rule for 'vel', the disjunction. The disjunction, where 'P' and 'Q' are both true, 'P or Q' is also true. Where 'P' is true and 'Q' is false, 'P or Q' is also true. Where 'P' is false and 'Q' is true, 'P or Q' is also true. It's only false, in fact, where they're both false.

Now, that’s represented on the tableau rules as a branching rule, and you’ve got ‘P’ on one side and ‘Q’ on the other.

I know that you're all thinking, “Why isn’t there a third branch with ‘P’ *and* ‘Q’ on it?” Can anyone tell me why there isn’t a third branch with ‘P’ *and* ‘Q’ on it?

Female: You only need ‘P’ *or* ‘Q’ to be true. You don’t need both to be true.

Lecturer: Well, except it *is* true when both are true.

Female: But you don’t *need* both to be true.

Lecturer: Good. That’s right. Because actually this enables me to put either ‘Q’ *or* ‘not-Q’ on here (the LH branch), and this enables me to put either ‘P’ or ‘not-P’ on there (the RH branch).

In fact we’ve captured the truth conditions of that in a branching rule, just with ‘P’ on one side and ‘Q’ on the other side.

I see somebody has written on this screen. Isn’t that appalling?

Male: It’s not visible.

Lecturer: Not for you it isn’t. It is for me.

	You can get to the tableau rules from the truth table definitions, which if you think about it isn't surprising: they are the same thing. (Slide 22) On page 20 of your handout booklet you will find all the rules, and you will also find the *incorrect* rule. I haven't put the truth table definition in, but you've got that earlier in your handbook, if you want to check out each against the other. I haven't put the rules down, but if you look in your handout booklet, which is – which page is the error on?
Male:	Page 21, top right.
Lecturer:	If you look at page 20, you will see that the tableau rules are all put there. There's the negation rule, the conjunction rule, the negated conjunction rule, and so on. But in the negated conditional, this is actually the negation rule. So if you have something like this, it's a stick rule, not a branch rule. It comes down, and you have 'P' and 'not-Q'. (Slide 23) Let's do a very simple tableau together. Would you like to produce the counterexample set for that tableau? Okay, counterexample set? Here it is. (Slide 24) Everyone have it down the page?
Female:	No.
Lecturer:	Bad girl.

Female: I know. I told you I was going to struggle with this.

Lecturer: What we've done is we've put the formulae on the left-hand side of the sequent, and then we've negated the formula on the right-hand side of the sequent, and put them down the page. That's the counterexample set of the sequent with which we started.

So the second step is we start applying the rules.

(Slide 25) Now, here you can't apply a rule to this ('P') or this ('~Q'), because they're both simple formulae. That's actually not simple, it's negated, but because it's only one negation it's considered a simple formula.

You can't analyse these any further, but that one you can analyse. You can apply a rule to it. So there's only one rule to apply: it's the rule for the conditional. Tell me whether it's a stick or a branch. Put your hand up if you have the answer. Until everyone has their hand up I'm not going to let you speak to me. Is it a stick or a branch: the rule for the conditional?

Female: A branch.

Lecturer: This is a stick (going straight down) , and this is a branch (two lines branching off). See?

Female: So stick means it must be and a branch is options?

Lecturer: Well, yes. Here (the stick) there's only one world will give us the truth conditions. Here (the branch) we need two worlds to give us the truth conditions.

Mary, is it a stick or a branch?

Mary: Branch.

Lecturer: It's a branch, good.

(Slide 26) Here we go. The conditional rule is a branching rule, and if you thought doing the formatting on truth tables was awful, this formatting was just terrible. I was tearing my hair out. So I hope you appreciate it.

We've got it's a branching rule, not a stick rule. What goes on the left-hand branch? What do I put on the left-hand branch here? Put your hand up when you know, so I can see. Right up so I can see who has got it.

This is not rocket science. All you're doing is looking at the rule on page 20 or 21, and you will be able to tell me what goes on the left-hand branch.

Sam?

Sam: 'Not-P'.

Lecturer: 'Not-P'. So we've got 'not-P' on one branch. Good. There we go. We put the negation of the antecedent. So we have a conditional here. We put the negation of the antecedent on the right-hand side.

What do we put on the left-hand side? Ooh, I'm sorry. (Slide 27) We now have a branch in which we find both ‘P’ and ‘not-P’. So the ‘not-P’ that’s generated from that is inconsistent with the ‘P’ that’s the second.

So we can actually close that. That is a *failed* attempt to produce a possible situation in which all the sentences are true together. So we can close that off. That is not a possible situation, and we show that by closing the branch: that’s a closed branch. It is not yet a closed *tableau*, because we still have an open branch, but it’s a closed branch. It’s a failed attempt to draw a situation where the premises and the negation of the conclusion are all true together.

Everyone understand that? Yes?

(Slide 28) What should I put on the right-hand branch?

Female: ‘Q’.

Lecturer: ‘Q’, good. On the right-hand branch goes a ‘Q’. Here we are.

(Slide 29) But look, we’ve got ‘not-Q’ and ‘Q’, so we close that as well. That’s another failed attempt to draw a possible situation in which the premises and the negation of the conclusion are all true together.

So we now have a closed tableau. Not just a closed branch, but a closed tableau. We’ve run out of branches. They're all closed. What that tells us is that the argument is...?

Male: Valid.

Lecturer: Valid, that’s right. The sequent is correct. The argument is valid because the counterexample is inconsistent.

Do you see we have a closed tableau there? Good.

(Slide 30) Our completed tableau tells us that every attempt to draw a possible situation in which the sentences of the counterexample set are all true together has failed.

Each rule – application of the rule – is a representation of the truth conditions of that sentence. So if we look at the – no, I'm not going to bother you with that. Let’s just mechanically do the rules.

(Slide 31) The sentences of the counterexample set are inconsistent, so the argument claim is...?

Female: Valid?

Male: True.

Lecturer: No. The argument claim is...?

Female: True?

Lecturer: No.

Female: Valid.

Lecturer: *Correct*. And the argument is...?

Female: Valid.

Lecturer: Valid, that’s right, yes. (Slide 32) Well, ‘true’ I put (laughter) Okay, you can have ‘true’. As I've said it, you can have ‘true’.

Male: You said earlier on that when you're doing the syntactic ones it doesn’t involve truth?

Lecturer: No, it doesn’t, but when you're teaching it, it’s much easier to use the word ‘true’. What it’s actually saying is the set consisting of that, and the negation of that, is closed. But the impact of that is that you know that the argument is valid.

(Slide 33) Let’s do a complex sequent. Now, here’s a nice one, isn’t it? Here’s a complex sequent. Will you all write down the counterexample please?

Male: Why is the arrow in between?

Lecturer: Why is the arrow in between? Ah, here you mean?

Male: And the first one.

Lecturer: What we've got here is much more complex formulae than anything you've dealt with before.

Do you remember yesterday we talked about the importance of brackets and how brackets disambiguate things? What brackets tell us is the scope of all the truth-functors, the truth-functional connectives.

So here, which is the scope of this arrow (the first arrow in the LH formula)?

Male: 'P' and 'Q'.

Lecturer: Just 'P' and 'Q', because the arrow is here, around this.

But what's the scope of *this* arrow (The second one in the formula)?

Male: Two sets of brackets.

Male: Everything that's in the...

Lecturer: So this is saying, "If that **(P → Q)** then that **(S & R)**,." Are you with me? Because the brackets pertaining to this arrow (the third arrow) are which ones?

Male: The outer ones.

Lecturer: The outer ones, exactly so. So the scope of this conjunction is that (R & T). The scope of that conditional is that **(~ P → ~ S)**. The scope of this conditional is that (**(R v T)** → **(~ P → ~ S))**.

Can you tell me here, what's the scope of this disjunction?

Male: 'R' and 'T'.

Lecturer: 'R' and 'T', because the brackets are here.

What's the scope of this conditional?

Male: 'Not-P' and 'not-S'.

Lecturer: 'Not-P' and 'not-S'. What's the scope of this conditional?

Female: The outer brackets.

Lecturer: The outer brackets. So this is, in effect, saying that 'if R or T, then if not-P, then not-S.' 'If R or T, then if not-P, then not-S'. That is what it is saying.

You would be able to read that all yourself, as long as you understand the import of the brackets.

So this argument claim is 'if P then Q, then S and R, if R or T, then if not-P, then not-S', together entail 'P'.

So there's no possible situation in which those two formulae are true (the formula on the LHS) and that formula is false (the

	formula on the RHS). Or as this is a syntactic sequent, the sets consisting of that formula, that formula, and the negation of that formula, 'R', is closed.
Male:	That's going to be a 32 line truth table, if you did it that way, won't it?
Lecturer:	Which is why we really don't want to do a truth table of that, because we've got five sentence letters, and that would generate a 32 line truth table. Can you imagine how boring that's going to be? Whereas this is actually going to be reasonably simple. Have you all done the counterexample?
Male:	Yes.
Male:	It's a 64 line.
Lecturer:	Oh, is it a 64 line? It's too many, anyway.
Male:	Two to the five …
Lecturer:	Far too many. (Slide 34) Let's create the counterexample set. Have we done that? (Slide 35) There it is – formula one, formula two, and the negation of the conclusion formula.

(Slide 36) So you've got to apply the rules to the formulae of this counterexample set.

Now, it actually doesn't matter in which order you apply the rules – sorry, to which formulae you apply the rules first. But actually we want a manageable truth table, and some ways of doing it might generate a truth table that has too many branches, and you really don't want one with six different branches if you can possibly avoid it.

As you gain in experience you will start to see which rules you can apply first to keep the truth table manageable, but you're not in a position to be able to do that at the moment, obviously, and you've got to apply all the rules to all the formulae. So just keep going.

Obviously I'm going to guide you in doing this.

(Slide 37) In applying the truth table rules to a formula, you apply them to the truth-functor with the smallest scope first.

Do you remember when yesterday we did the 'not-T arrow Q', or 'not-P arrow Q', we did the '*not'*, the scope of which was just the 'P' first, and *then* did the conditional? Well, we do it the other way around for a tableau. We apply them to the truth-functor with the largest scope. I will show you what I mean.

Male: The smallest scope?

Lecturer: (Slide 38) I've labelled each of the formulae so that we can track the way through down this.

Looking only at formula one, which is the truth-functor with the largest scope in formula one?

There are three truth-functors, aren't there, in that formula? There are two conditionals and a conjunction. Which is the one with the largest scope?

Female: The second one.

Male: The one in the middle.

Lecturer: The second is conditional, that's right. Because the scope of the conjunction is just the 'S and R', the scope of that conditional is just the 'P and Q', and the scope of that conditional is the whole formula. Are you with me? So that's the rule that we apply first. (Slide 39)

This is a conditional rule. Is it a stick or a branch?

Male: A branch.

Female: Why do we pick level one and not the second row?

Lecturer: I explained this a minute ago. Who can tell me why I've picked number one and not number two? Why am I applying the rules to number one and not number two to start with?

Male: One is simpler.

Lecturer: Because I know that it’s going to generate a simpler tableau. You're not in a position to really know that yet.

If you would rather do two first, you can do two first. Actually, it’s not going to make that much difference with this. It’s completely arbitrary which one you do first. I think I chose number one just because it was number one in this case.

Looking at number 1, forget the other formulae completely for the moment. Looking at number one, what’s the rule? Is it a stick rule or a branch rule? Is it a stick or a branch?

Female: A branch.

Male: A branch.

Lecturer: It’s a branch. All conditionals are branching rules. So I've put the branching in.

What do I put on which branch? Now, work that out for yourself, and then put your hands up when you’ve got it.

Don’t forget you’re blind to the complexity. All you're doing is following the rules at the moment.

Dermot, what goes on the left-hand branch?

Dermot: (Not (P and Q)).

Lecturer: Sorry, say that-

Dermot: Not that first branch.

Lecturer: Good, yes, that’s right. (Laughter) That first formula, but negated, is what you mean. Good. Well done.

Sorry, you said (not (P *and* Q)), but that’s not an ‘and’ is it, so that was wrong. I knew what you meant, but I had to get you to say the right thing.

So I've taken this out. I've given both, actually.

(Slide 40) So branch rule, and on the first side is the negation of the antecedent of that formula, and on the other side is just the consequent as it is.

One, two, and three are the sets of the counterexample set. Four, this line of four, is the result of applying the conditional rule to formula one. You can go back and track through how we’ve done this later on.

Do you see that we’re completely ignoring formula two and three at the moment? We’re just doing formula one.

It’s a conditional. The overall form of formula one is a conditional. What we’re doing is a branch rule, and on the left-hand side we put the negation of the antecedent of the conditional, and on the right-hand side we put just the consequent of the conditional.

That gives us our second lot of formulae.

The formulae at four were achieved by applying the conditional rule to formula one.

Our next task is to apply the rules to the formulae generated. We could now ignore this and just go on to that, but then you're working with two branches, and really you want to try and keep – well, you won't be able to do this, but you want as few branches as possible. You will see why in a minute.

Our next task is to apply the rules to these formulae, the two formulae by formula one: both the left-hand and the right-hand branch.

What's the overall form of this formula?

Male: A branch.

Lecturer: No. I asked what the overall form of this formula is.

Male: Oh, it's the...

Male: Negated conditional.

Lecturer: It's the negated conditional. It's very important that it's negated. It's not just the conditional. It's the *negated* conditional. We don't use the conditional rule on that. We use the negated conditional rule, which is different. So is it a stick or a branch?

Male: It's a stick.

Female: A stick.

Lecturer: It's a stick, isn't it, the negated conditional rule? What do we put here then?

Female: (P and not-Q)?

Lecturer: (P and not-Q), that's right. On the stick, (P and not-Q). What's the overall form of this formula?

Female: A conjunction.

Lecturer: It's a conjunction. That's right. Is that a stick or a branch?

Female: A stick?

Lecturer: It's a stick again, and we put on it...?

Female: (S and R).

Lecturer: (S and R), good.

Female: [inaudible].

Lecturer: (Slide 41) Now, notice I didn't get as far as putting the 'not-Q' on there, because the minute I put the 'P' there it's inconsistent and I can close it. So that's good. We're left with one branch, which is even better. So I put (S and R) here, and I've only got one branch open.

What I would do normally is tick formula one. The reason I didn't do it is because I was getting so fed up with the bloody thing. (Laughter)

Tick formula one to show that it's done, and tick both formulas on level four to show that those are done, which means that we still know that we haven't yet done formula two.

(Slide 42) We've now got level five, and these branches come from applying the negated conditional rule to the left-hand branch of four. That one. The negated conditional rule to the left-hand branch and the conjunction rule to the right-hand branch.

See where we're getting all these things from? You can go back and work through this for yourself, obviously.

Male: So you don't need to close both branches, as long as you close one branch that...?

Lecturer: Well, I can't close that branch. There's no contradiction in there, is there? The only simple formulae that we've got on there is 'not-P', 'S' and 'R', and so far those are entirely consistent, aren't they?

Male: Because you've got one closed branch it's sufficient to tick formula one?

Lecturer: I'm ticking formula one not because there's a branch closed, but because I've applied all the rules to formula one. Once I've applied the conditional rule to formula one, and got these two, I can tick off formula one. Once I've applied the rules to these two, I can tick off both these two. So I tick off a formula once I've applied the rules to it.

So you've got two stick rules. That branch closes. The formula on the right-hand branch is a conjunction, which is another stick rule.

That's where we are. We have now got to go back to – oh, I have ticked it. Can you see a green tick up there?

Male: [inaudible discussion]

Lecturer: But there should also be a green tick there and a green tick there.

(Slide 41) We need to apply the rule to formula two now. So, looking only at formula two, we can forget formula one, what's the overall form of formula two, i.e. which is the truth-functor with the largest scope?

Female: Conditional.

Lecturer: It's the conditional. Which conditional? There are two conditionals.

Female: The first conditional.

Lecturer: The first conditional. It's that one isn't it?

Why is that? Because the scope of the vel, the disjunction, is those two, and the scope of that negation is 'P', and that negation is 'S', and the scope of that conditional is 'not-P' 'not-S', and the scope of that is the whole formula. So we apply the conditional rule to that formula.

Is it a stick or a branch? The conditional rule is that a stick or a branch?

Female: Branch.

Lecturer: It's a branch. So there we are. (Slide 43) It's a conditional rule, and we've got the branch. We're opening up another branch again, which is a bit of an irritant, but never mind. There's nothing we can do about it.

Which formula do we put on the left-hand side of this branch? Put your hand up when you've got the answer so that we can give other people a chance to get it.

Which formula are we putting? We're doing formula two. It's a branching rule, a conditional rule. What's the formula we put on the left-hand side? Anne?

Anne: (R vel T)?

Lecturer: (R vel T)?

Anne: Yes.

Lecturer: Well, is it?

Male: No, it’s (not-R vel T).

Lecturer: ‘Not-R’, that’s right, because the antecedent is negated, according to the conjunction rule. That’s right.

If you weren’t quick enough to see, what do we put on the right-hand branch?

Male: (If not-P then not-S).

Lecturer: The whole of that formula, just exactly as it is. We just put the consequent. So here is the result of that. (Not-R vel T), but just the consequent there. Six, which is this level, has come from applying the conditional rule to formula two.

You can go back and check. All the working is available here. Good.

Now, those are both complex, so there’s no closing anything here, because as long as a formula is complex it can’t be used to close anything. (Slide 44) So we’ve got to apply the rules to that and to that.

What’s the overall form of this formula, not vel, or not negated disjunction? Is it a stick or a branch?

Male: A stick.

Lecturer: It’s a stick. That’s right. Think of the English here. It’s (not-R or T). That means it’s both ‘not-R’ *and* ‘not-T’, doesn’t it? It’s not R *or* T’ becomes ‘it’s neither R *nor* T’. So that’s a stick, and you've got ‘not-R’ and ‘not-T’ on the same stick. Good.

What’s the overall form of this formula, a stick or a branch?

Male: A branch.

Lecturer: (Slide 45) A branch. It’s a conditional again, isn’t it? What goes on the left-hand side of the branch?

Male: ‘Not-not-P’.

Lecturer: The negation of the antecedent, so it’s ‘not-not-P’, that’s right.

What goes on the right-hand branch? Not Chris! Go on.

Female: ‘Not-S’.

Lecturer: ‘Not-S’, that’s right, exactly. The consequent goes on the branch.

Male: Sorry, I've got mixed up in the trees. Which is a stick and which is a branch?

Lecturer: That’s a stick (one stroke downwards) and that’s a branch (two strokes branching out).

Male: Oh, yes, indeed, but when do I use them?

Lecturer: You look at all the rules I've given you on page 20 and 21, and that will tell you whether it’s a stick or a branch.

So that’s a stick rule, and that’s a branch rule, and you’ve already worked it out, so there you are. (Slide 46) We can put ‘not-R’ and ‘not-T’ should go there, and ‘not-not-P’ and ‘not-S’, but have you noticed something important?

Male: You can close that one.

Lecturer: You can close that one. You’ve got ‘R’ and ‘not-R’ on that branch, so that’s closed.

Male: You've got ‘P’ and ‘not-P’ on the next one.

Lecturer: You've got ‘not-P’ and ‘not-not-P’ on that one, so you can close that one off. You've got ‘S’ and ‘not-S’ on that one. So actually, at that level, we can close the lot. All the branches are closed, and the argument claim is correct or true. (Slide 47)

Female: Valid.

Male: No.

Lecturer: And the argument is...?

Male: Valid.

Lecturer: Valid, well done. (Slide 48)

Male: Marianne, could you do any manipulation on the (not -R or T)? Does that come out to (not-R and not-T), straight away?

Lecturer: Yes. You look at the negated disjunction rule. So ‘not-R or T’ becomes ‘not-R’ and ‘not-T’, but the minute you write ‘not-R’ there you can close it. Why bother putting the ‘not-T’ in? Although if you do it doesn’t matter, as long as you go back and then see that in each of the...

Each of these branches is a failed attempt. Each time we apply a rule, what we’re doing is we’re representing the truth conditions, the conditions under which these formulae are true and false, and by putting them all together we’re trying to find out whether there are *any* conditions in which all the formulae are true together, and we’ve failed! There isn’t such a situation here. Each branch has closed. So we know that the set is inconsistent, and that therefore the argument that generated the set is valid.

Male: Can you just...?

Lecturer: Yes?

Male: Branch five, why did you close it off?

Lecturer: Why did I close that one?

Male: Yes, because it's …

Lecturer: Because there's 'not-P' up there. That's the counterexample, so from one, two, and three, and from formula one we generated, by a branching rule, the conditional rule, those two. Then by the *not* conditional, we generated that, and that contradicts that. That contradicts that. That contradicts that. That contradicts that. So there isn't an open branch.

Male: When you do the breakdown, it looks as though there's a hierarchical decomposition, but it's not. When you get down to tilde bracket not R vel T bracket, that branching, that branching is stemming from equation two.

Lecturer: When you say, "This branch", do you mean this branching?

Male: Yes, that branching is stemming from equation two.

Lecturer: From formula two, yes.

Male: Formula two, yes, but it is diagrammed as though it was stemming from, linearly, the single formula down there.

Lecturer: SR? But that's because this gives us – so at four we've shown the possible world in which formula one is true.

Male: Indeed.

Lecturer: There are two possible worlds consistent with that being true. That's the first world. That's the second world.

Both of these are complex, so we've then got to apply the rules to those, and we discover that that actually *isn't* a world in which formula one is true consistently with formula three. That's still a world – this is one world in which formula one is true.

So, if you think about the English again, ((if P then Q), then (S or R)), that's true in the world where 'not-P' and 'S and R'.

Male: I've no quarrel with the logic and the analysis, but diagrammatically that is representing the formula, equation two, as though it was descending from a branch out of …

Lecturer: Yes, and the reason for that is we want to check the truth conditions of all these formulae against each other. Each branch is an attempt to draw a possible world where those three formulae are true together. So we've got to combine … so, just as in the truth table, we combine the truth values of all the formulae.

In the tableau we're also combining the truth values of all the formulae, but we're doing it differently. We're doing it according to tableau rules instead of the truth table rules.

Male: But unless you write some description, as you have done on the left, to show – for example – that the decomposition is coming from a certain area, diagrammatically it's inconsistent with any other decompositions where you have a tree structure. It's not a tree structure.

Male: It's almost like a grafted fruit tree, isn't it, where you've got the root stub is different from the ...? (Laughter)

Lecturer: I'm lost now. (Laughter) I understand what you're ...

Male: If you've got the line: tilde bracket R vel T bracket.

Lecturer: Tilde?

Male: Yes. If you take that horizontal line – no, horizontally.

Female: The other way.

Lecturer: Okay, yes.

Male: Across there. That line does not descend from the R above it.

[inaudible discussion].

Lecturer: No, it doesn't. That comes from formula two.

Female: It's not a tree.

Male: In all the other you have a linkage to show the hierarchical decomposition. That's doesn't show.

Lecturer: I'm going to have to stand on this chair. What we're trying to do is to show whether there's any possible situation in which formulas one, two, and three are true together.

We've got to look at the truth conditions – the conditions of truth and falsity of *that* formula, together with the truth and falsity conditions of *that* formula, together with the truth and falsity conditions of *that* formula.

So what we're doing is first we are getting the truth conditions of *that* formula. There are two possible worlds in which that's true. This is a description of each of them. We get that down to individual sentences, single sentences.

By this point we have completely represented the truth conditions of that, and we see that there's an inconsistency between formula one and formula three.

This is not a possible world in which formulas one and three are true together, but so far this is *still* a possible world in which formulas one, two, and three are true together. This hasn't been closed off.

We now want to get out the truth conditions of formula two, and the two possible worlds in which formula two is true. We want to combine the truth conditions of formula two with the truth conditions of formula one, so we generate the tableau for formula two from the branch from formula one, and that's what we get. Are you with it?

Male: The thing is here that you can apply the rules to either one, two, or three in any order you like. You choose the best order you think it's going to work out at, and all you're doing is you're adding on your analysis of formula two at that point there, but …

Male: … The analogy is the parent/child relationship …

Male: Maybe we want some way of representing that, and you could add a way of representing [cross talking].

Lecturer: What do you mean by parent/child relationship?

Male: You are doing an analysis of a relationship where you get that line …

Lecturer: What do you mean by ‘the parent’?

Male: Two. Two is the parent of the next step of that.

Lecturer: Yes, that’s right.

Male: Now, there is no visual representation of that parent.

Lecturer: Yes, there is.

Male: Where?

Lecturer: It’s because the stem of the tree is here. Those three formulae there are the counterexample one, two, and three. Four is the level of one, coming out of the truth conditions of one. Five – I think that’s five, is it, or six – is two coming out. So the parent is actually better seen as those three, not as any one of those.

Male: I think I know what you're saying, because in an organisational chart – I'm just thinking with you – it’s not flowing from ‘R’. It’s flowing from ‘Q’. But in an organisational chart you would read that as ....

Lecturer:	It’s flowing from these three, not from ‘R’.
Male:	If it’s a more complex representation of a situation, then I think you need the parent/child relationships. You do in any hierarchical breakdown, which is actually what that is.
Lecturer:	It’s not a hierarchy. I'm a bit worried about what you mean. There’s no hierarchy. I'm not doing that because it’s important to do number one first.
Male:	Something descends from something else. You’ve got line two [cross talking].
Lecturer:	But you think that that’s descending. All these lines here, all these branches here, are descending from that counterexample set.
Male:	If you didn’t come with the context, that’s what you would think, is that it descends.
Lecturer:	I have to say, I don’t really understand the objection. That is true. I don’t.
Male:	It’s just visually. I'm with you – I don’t believe it’s a hierarchy – but it’s visually.

If you had nineteen points on the way down, and then you had to go back and break down point two into point six there, but there were nineteen points ahead of it, it gets very confusing. Where do I go back to?

Lecturer: But the thing is, when you practice it's not confusing, when you know what you're doing.

You don't have to have all this. I'm putting this in because you're beginning, and I'm trying to teach you how to do it. I don't need that. I can see exactly where all these things are coming from, because I know the rules.

When you don't know the rules, it's confusing. Isn't that always the case?

Female: Yes.

Male: I think I've seen in some book whereby they would do say a dash line from line one down to line four, to show that on line four you're analysing … then another line from line two down to line six.

Lecturer: I find that very confusing, but yes, I've seen that. So instead of doing this, I could draw a line from there to there, and a line from there to there, and I just find that – we don't really need more lines.

Male: I think the whole point is once you have got to that section, on line five as you've got it, where (S and R) are on the right-hand side, you can't do any more with those. It's not closed, but you can't develop them any further.

Lecturer: You've reduced that formula one to atomic formulae, to simple formulae. There's no other rule you can apply.

Male: One side is closed and the other side is still open. So now you've got to look back at the top and take your next example, because that's still a part of the possible world.

Lecturer: Well, it's still a part of that counterexample set, which is ___ [cross talking].

Male: Yes, but you've got that world still open.

Lecturer: Listen, we are supposed to be finishing now, so do you mind if I move on, because I don't know what we're doing next? Oh, that's it. (Laughter) (Slide 49) That is it. So that's alright. We can carry on arguing, if you like. (Laughter)

So we've finished that really complex formula, and I don't mind at all going through another one in the question and answer session, and seeing if it becomes clearer.

Male: I was just wondering, for those who don't like the style of it, would it help to have 1, 2, and 3, and then 1A, 1B, 2A, 2B?

Lecturer: Yes. Actually, I used to do this on blackboards. Apart from the fact that blackboards were so ghastly, it's much, much easier to do it on a blackboard.

If you like we will start, after the coffee break, with doing one on a flipchart, so I can show you just much more quickly how these things happen, because it's a bit difficult. It's not obvious getting these bloody branches in the right place. Sometimes technology is not a...

Male: Well, I can answer that!

Female: Is it simpler to think of the stick as being the end of the road that is the atomic structure? That the branching will always lead...?

Lecturer: No, because that's not the end of the road, is it? That's a stick, but it's not the end of the road.

Female: No, but it's the end of that particular road. You you said you can't apply any more rules to that.

Lecturer: Well, only because these happen to be simple formulae. They could be complex. So this might be (P and Q) and (S or R) or something.

Female: But they're not, are they? (Laughter)

Lecturer: They're not, but they might be.

Female: Well, yes, but in that case …

Lecturer: No. The stick does not signal the end of the road. This is the only thing that signals the end of the road, and that signals the end of the road because we’ve got a formula and its negation. So it’s a contradiction.

Female: So you're saying that a formula could actually be negated, that it would actually be proven to be closed: a whole formula?

Lecturer: If there’s a formula and its negation it *is* closed, end of story. There’s no ‘if’ about it.

Let me just finish. We’ve done that one. (Slide 50) A couple of syntactic sequents for you to practice at home, and the answers of those are on page eight, so you can check them. So those are two to practice at home.

(Slide 51) Here I'm telling you that all of these are correct, which means that in doing them you know that they should close. So if they don’t close, when you're finding the answer, you’ve got something wrong, but I've still put the answers there in the answer booklet.

That’s it folks. How is that? We’re one minute over. In the next session we can do whatever you like, because it’s an hour and a half of looking at the ...

Male: Paperwork.

(Applause)