A romp through the foothills of logic – Session 3

It would be a good idea to watch the short podcast ‘Understanding Truth Tables’ before attempting this podcast.

Lecturer: (Slide 2) In the last session we learnt how to formalise English arguments as sequents of the propositional calculus. Who can tell me what a sequent is?

Male: Something in sequence – one, two, three, four, five, six, seven?

Lecturer: No. What is a *sequent*?

[inaudible discussion]

Lecturer: No.

Male: Numerical order?

Lecturer: No. What is a sequent?

Male: A set of statements?

Male: Something used to make a statement?

Lecturer: No, none of these things. It’s an *argument claim*.

There are semantic sequents and there are syntactic sequents, and each of them is an argument claim.

The claim that it makes is of course the claim of any argument, which is that ‘these premises entail this conclusion’.

Female: So is that a conclusion that you're talking about …?

Lecturer: No, I'm talking about the *sequent*, which is the argument claim, which is the formalised claim that the premises entail the conclusion.

Female: So that encompasses the whole thing?

Lecturer: Indeed. So we learnt how to formalise an English argument. An English argument says that ‘this conclusion follows from these premises’, and we formalise that as a sequent of the propositional calculus, and each of the sequents tells us that these premises entail this conclusion.

In other words, there’s no possible situation in which these premises are true and this conclusion is false.

We saw that there were two types of sequents – syntactic and semantic. What’s the difference between the two? This is revision for you …

[inaudible discussion]

Male: One comes from the grammatical form, from syntax, syntactic, and semantic is from semantics, the meaning of the words, the words themselves.

Lecturer: It’s true the semantic sequent has to do with meaning and truth, and the syntactic sequent has to do with grammar and structure, but one of them was defined in terms of – can you tell me how a semantic sequent is defined, anyone?

Female: There is no logical something? (Laughter)

Lecturer: Almost. You're doing very well.

Female: No logical reason?

Lecturer: Not quite.

Male: Which one? Sorry, which one did you say?

Lecturer: Semantic we’re looking at.

[inaudible discussion]

Lecturer: Start again.

Female: No way in which you can have something on the left that isn't true and still keep the other side the way it is? I don't know …

Lecturer: You're in the right direction. The semantic sequent is defined as 'there is no logically possible situation in which the formulae on the left-hand side of the sequent are all true and the formulae on the right-hand side are false'.

The syntactic sequent, how is that defined?

It doesn't take long, does it? It's dissolved in a glass of wine. (Laughter)

Female: Grammar and structure?

Lecturer: Something close … good.

Male: The set of things on the left … taken together with *not* the set of things on the right is a closed thing.

Lecturer: Very good. Almost, yes. 'The set consisting of the formulae on the left ….'– except we don't talk about that.

'The set consisting of the counterexample of the argument is closed'. That's what it is. The counterexample consists of the premises plus the negation of the conclusion. We will talk more about counterexamples later.

I think that's quite enough revision. (Laughter)

(Slide 3) In this session we're going to learn how to test semantic sequents by using truth tables, whereas in the session tomorrow we will learn how to test *syntactic* sequents by means of truth trees, or 'tableau', they're called.

So how do we test semantic sequents by means of truth tables?

Now, I'm being told by Chris not to stand in front of this, because it makes me look ghastly. (Laughter) So obviously I'm going to stay here the whole of the rest of the session.

(Slide 4) You've already met the truth table definitions of the truth-functional connectives of propositional logic, so 'P and Q'. Remember the truth table – which you all came up with – I didn't have to tell you what it was … that's what the truth table is.

That gives you the definition of 'and', because it gives you the truth value in every possible situation of 'and'.

(Slide 5) Truth tables are tabular representations of all the possible situations generated by the combination of truth values.

You've got 'P' and 'Q'. Each of them can be either true or false. So there are four possible situations: four possible combinations of truth value. Because these are truth-functional connectives, each of the rows will have a truth value in it. That truth value will give you the truth conditions for the whole formula.

(Slide 6) Truth tables enable us to check whether it really is the case that there's no logically possible situation in which the formulae on the left-hand side of the sequent are all true, and the formulae on the right-hand side of the sequent are false.

We use truth tables to check whether the argument *claim* – the claim that the conclusion follows from the premises – is a correct claim.

(Slide 7) To evaluate semantic sequents we need to have to hand the truth table definitions of all the truth-functional connectives, and luckily you have that on your handouts on page 18 to 20.

(Slide 8) There they are again. What I've done here is I've put them into one table, so ‘P and Q’ – whoa, I'm standing in front of it, but never mind. (Laughter)

This one’s only got two in, because this is a unary connective. So where ‘P’ is true, ‘not-P’ is false. Where ‘P’ is false, ‘not-P’ is true. ‘P and Q’, where they're both true, it’s true. Where one’s false, it’s false. Where one’s false, it’s false, and so on.

That’s exactly what I told you earlier, except I've put them together on one table, which makes it easier for me. But you’ve got it on your handouts as well, because I won’t be able to show you this when we’re working through it. You will need to work from your handouts.

(Slide 9) So we’re going to learn how to evaluate semantic sequents by evaluating this very simple sequent. You only have to look at that to see whether it’s valid or not, don’t you? Is it valid?

Male: Yes.

Lecturer: Yes, that’s valid. Because this means there is no logically possible situation in which those are both true and that’s false, and you can see immediately that if those are both true then that has to be true.

We're working on a sequent where we know the answer to the question 'is it correct or not?' So that's quite useful.

First we've got to draw the empty truth table. To evaluate that sequent you need a truth table. (Slide 10) You need to draw an empty truth table, and you need to know how many columns and how many rows to draw.

Now, you've seen me doing it, and none of you have questioned how many rows and columns that I'm putting on, and that's because it is actually quite obvious.

For the number of columns you need a number of sentence letters. How many sentence letters are there in that sequent? Two – 'P' and 'Q'. Plus the number of formulae in the sequent. How many are there in that sequent? Three – one, two, three. Two plus three is five. Plus a column for the sequent itself. So we need six columns.

(Slide 11) Here you are – six columns. So you've got each sentence letter, plus one formula, two formula, three formula, plus one column for the sequent itself. Are you with me?

Male: Yes.

Lecturer: This is just the mechanical drawing of the truth table.

(Slide 12) To determine the number of *rows* we need to look at the number of sentence letters.

If there are two letters we need four rows. If there are three letters we need eight rows. If there are four letters we need sixteen rows and so on.

That's because you're trying to show every possible combination of the truth values of the letters.

So how many rows do we need for the sequent that we've got here?

Male: Thirty two – no, sixty four.

Lecturer: No. You're jumping to conclusions here. We've got two sentence letters, so how...?

Male: Four.

Lecturer: (Slide 13) We need four rows. That's all. So we need a truth table that – where's the one that worked?

Male: There are a couple of them on the wall.

Lecturer: Good. I've got one here that works. So we need six across this way, and two of them can be quite small, and then we need one, two, three, four, five, six, and we need four across, like that.

That's because we need true, false, true, false, true, true, false, false. That's the way you always do it.

The next one down is true, true, true, true, false, false, false, false. Then you get all the combinations.

What do I put in here? ‘P arrow Q’, and then ‘P’, and then the sequent itself, and then ‘Q’.

So that’s the empty truth table that will enable us to test that sequent. Any questions from that?

Right, there you are. There’s the empty truth table. (Slide 14) Now we’ve got to complete the truth table according to the rules.

First we’re going to do the really easy columns. So, in a world where ‘P’ is true and ‘Q’ is true, what’s the truth value of ‘Q’?

Female: True.

Lecturer: Dead easy, wasn’t it? In a world where ‘P’ is true and ‘Q’ is false, what’s the truth value of ‘Q’?

Female: False.

Lecturer: You've all got this. What’s the next one?

Female: True.

Female: False.

Lecturer: It's exactly the same, in other words. All you've got is 'Q', so you just transfer the truth values from 'Q' to that.

We can do the same with 'P'. What do I put in here?

[inaudible discussion]

Lecturer: True, true, false, false. You've got the idea immediately. (Slide 15) So you see we've got here true, false, true, false, exactly the same, and true, true, false, false. Anyone not with me on that?

(Slide 16) Next we do the more complex formulae. We've got the sequent left, but we will do that at the end. We can't do that until we've got to the end.

Here we've got a complex formula, and we need to fill that in according to the truth table definition for conditional. So make sure you've identified, on your truth table definitions, the formula for the conditional. (Slide 17)

Female: I have a question about the formula for the conditional. Is it supposed to be assumed at the moment or … [inaudible]?

Lecturer: Yes, you are just assuming it. That's the one that you've got to take on my authority for now, though you're quite right to question it. It's not obvious where it comes from at all, but believe me, that's it. We haven't got time to …

Female: … I figured! [Laughter]

Lecturer: We've got a whole session tomorrow for questions and answers, so that would be a very good question to ask during that.

So, given the truth table definition of the conditional, what do I put in here? Can anyone tell me? What do I put here in row one?

Male: True.

Lecturer: Does anyone not understand why I'm putting 'true' there?

Male: Should you not use small letters like the other ones?

Lecturer: Yes, I probably should. You're quite right. I don't need to, actually. Usually I say uppercase letters for whole formulae. These are whole formulae. But I've started like that so I will finish.

Does anyone not understand why I put 'true' there? No? Okay.

Look at the truth table definition of the arrow, which is this one, this layer here. Where 'P' is true and 'Q 'is true, 'P arrow Q' is ...?

Female: True.

Lecturer: True. That's why I put 'true' there.

What do I put under row two, where I put ‘P is true’ and ‘Q is false’?

Female: False.

Lecturer: False. What do I put under ‘P is false’ and ‘Q is true’?

Male: True.

Lecturer: True. And false, false...?

Female: True.

Lecturer: Is true. So it’s false, false, true. I'm just copying from the table. There’s nothing odd about this.

The truth table for ‘P arrow Q’ tells us that that’s true unless the antecedent is true and the consequent is false. I will justify that tomorrow if you ask me in the question and answer session.

(Slide 18) Does anyone not understand where I've got all those truth values?

Where it would be harder is if this was ‘Q arrow P’. How would I work it out if this was ‘Q arrow P’ instead of ‘P arrow Q’? Can anyone tell me?

Male: Just swap the two around? Swap P and Q in the truth table that defines the arrow here …?

Lecturer: Yes. You're absolutely right. What you do, so 'P arrow Q' is P is here, true, and Q is true. It's true. If it were 'Q arrow P', it would actually be the same. You would have true and true. But if it's 'Q arrow P', instead of true, false, and therefore false, you would have false, *true*, and therefore...?

Male: True.

Lecturer: True, yes. Do you see? So you have to be very careful, in filling these in, that you've got the right sentence letter in the right place.

This is an easy one, because I've done it in the same order as the truth table definitions are giving you.

Let's see where we are. Okay, so that's where we are. We've done the truth values for all the formulae now. (Slide 19) The only thing we've got left to do is to check whether the sequent itself is correct or incorrect.

So there we're not looking at whether it's true or false, we're looking at whether it's correct or incorrect, because the sequent says there is no logically possible situation – and each of these is a logically possible situation, okay? (Slide 20)

There is no logically possible situation in which all the formulae on the left-hand side are true *and* the formulae on the right-hand side false.

Well, is there a situation where all the formulae on the left-hand side are true?

Female: Yes.

Lecturer: Which one?

Female: The top row.

Lecturer: (Slide 21) The top one, both of the formulae on the left-hand side of the sequent are true, aren't they?

Male: Yes.

Lecturer: Everything else is actually of no interest to us, because it's not the case that the formulae on the left-hand side are all true. So it's only that one that's interesting.

Is it the case that all the formulae on the left-hand side are all true and the formulae on the right-hand side are false, or not?

Female: No.

Lecturer: It's not, is it? We'll carry on.

(Slide 22) So we're looking at number one. That's the only one that interests us.

In row one, if the formula on the right-hand side is false then the sequent is incorrect. (Slide 23) But it's not false, is it? It's true. Therefore the sequent is correct, and we tick it, and we see that that sequent is a correct sequent.

There's no possible situation where the formulae on the left-hand side are true *and* the formulae on the right-hand side are false, therefore the sequent is correct, and we know that the argument is valid.

(Slide 24) Would you like to test that sequent on your own?

Male: Just to check, you're not interested in any of the other results?

Lecturer: No.

Male: Because the other results don't apply?

Lecturer: Because it's only if all the formulae on the left-hand side are true that we're interested in the formulae on the right-hand side.

None of these situations is such that all the formulae on the left-hand side are true. One is true and one is false in each case, and that's therefore not interesting.

So we're only interested in the ones where all the formulae on the left-hand side are true.

So do that one on your own. Let's see if you can do that on your own.

Male: That's presumably why you rewrite the 'P' column?

Lecturer: Why we write the...?

Male: The 'P' column is repeated, but the second incarnation of the 'P' column is there to allow you to make the comparison easily, without getting lost?

Lecturer: Yes.

Male: Suppose you just [inaudible]-

Lecturer: Because that's a sentence letter and that's a formula. That's a sentence letter and that's a premise, if you like.

Male: Yes, okay.

Lecturer: That just gives us the key to the truth value, and that gives us the actual truth value. They happen to be the same, because they happen to be the same formula.

See if you can do that one. Put up your hands if you need my help.

(Slide 25) Just draw the truth table for now and don't start completing it. I mean complete the truth value to the sentence letters, but don't do the formula yet.

Put up your hands if you want my help.

So how many columns do I need?

Male: The same number as last time.

Female: Six.

Lecturer: Six? Okay, so one, two, three, four, five, six.

How many rows do I need … [inaudible discussion]? How many rows do I need [apart from that]?

Female: [Four].

Lecturer: Four? Okay, one, two, three, four. What do I put in here? P, Q. Then the truth values down here?

Male: TTFF.

Lecturer: TTFF and TFTF, good. Then what do I put here?

Female: ‘P then Q’.

Lecturer: ‘P then Q’.

Male: [inaudible discussion]. Oh, sorry.

Female: Oh, sorry.

Lecturer: No. All we're doing at the moment is the truth table, the empty truth table.

Male: 'Q'.

Lecturer: 'Q'.

Female: Sequent.

Lecturer: Sequent.

Male: 'P'.

Lecturer: 'P'. Good. (Slide 26) So that's the truth table. That's the empty truth table. We haven't started proving anything yet.

Is that what people had?

Male: Can you put an F on the bottom row of Q, TFTF?

Lecturer: Sorry. You're quite right. Good.

(Slide 27) The next thing we do is we start proving it. The truth table should be very like the one we drew for the last sequent. In fact, it’s identical to the one we drew for the last sequent, except it’s got a different sequent in it.

So your next steps, put in the appropriate truth values for the simple formulae, the complex formulae, and then for the sequent.

Are we ready? Would people like more time or should we get on?

What do I put under here?

Male: TTFF?

Lecturer: No, Chris, you know how to do this. Let’s have someone else. What do I put under the ‘P’?

Female: TFF.

Lecturer: TTFF, exactly the same as under ‘P’ there, exactly the same. What do I put under the ‘Q’?

Male: [The same].

Male: TFTF.

Lecturer: TFTF, because it’s exactly the same there, that is just ‘Q’. So what’s the truth value of ‘Q’ in the world in which ‘Q ‘is true? It’s true, obviously. What goes in here?

Male: TFTT.

Lecturer: Who said that? Steve? TFTT.

Small cases again, but never mind. It doesn’t matter.

That comes from the truth table definition of the conditional again.

Now we get to check it. Looking at each of these rows separately, because each row represents a different possible world, is there a possible world in which the premises are all true – in which the formulae on the left-hand side of the sequent are all true?

Female: The first one.

Lecturer: The first one, so that one. Is there any other one?

Male: Yes.

Lecturer: Which one?

Male: The third.

Lecturer: Three is also such that both the – sorry, it's not obvious, but that's because of my writing …

But 'if P then Q' is true, because 'P' is false and 'Q' is true, therefore it's true, 'Q' is true, therefore that one, the formulae on the left-hand side, are also all true.

(Slide 28) Now we need to check whether there's a situation in which they are all true on the left-hand side and yet false on the right-hand side. Is there one?

Male: Yes [inaudible].

Lecturer: Well, that's not one, is it, because that's true on the right-hand side as well? But that one is a counterexample.

Male: Sorry, I've missed something here. I thought 'P' was false on number three?

Lecturer: You thought was 'P' was false on number three? It is.

Male: Yes.

Female: I made exactly the same thing. You’re not supposed to count when you're looking at it. You just count the formulae bit. So this bit, ‘if P then Q – T’, and then the ‘Q’, and you don’t count the ‘P’ and ‘Q’ initially.

Lecturer: These are just the key. These two both tell you *only* what the truth values are. They're telling you all the different combinations.

Given that you’ve got two truth values, you need a world in which both are true, both are false, and one of them is true, one of them is false. That’s the same on all of them.

In the world in which ‘P’ is false, then ‘P’ is false. So the formulae on the right-hand side is also false.

Here you’ve got a situation where both the premises are true, and the conclusion is false, and therefore that’s a counterexample to the argument.

Does that make sense? This ‘P’ is very different. This ‘P’ (the one on the RHS of the sequent) is a conclusion, and this ‘P’ (the one in the key) is actually just the key.

This ‘P’ is just telling us the different truth values, in order to give us the world that we’re looking at, in order to determine whether the sequent is true or false *in that world*.

So all we want is one possible world in which all the premises are true, and the conclusions false, (Slide 29) and we know that that sequent is incorrect, and therefore …

Male: So it means the whole sequent is incorrect?

Lecturer: It means the whole sequent is incorrect.

Male: So you could have three true, three-

Lecturer: Remember the definition of validity is there is no possible world in which all the premises are true and the conclusion false.

These are all the possible worlds. So what we're saying is there is *no possible world*.

So one world in which the premises are true, and the conclusion false, is enough to show that the whole thing is invalid.

Male: So if you had a table of 64 entries, and you only one cross somewhere-

Lecturer: One cross shows you the sequent is incorrect, the argument is invalid, yes.

Male: Ah, right.

Lecturer: It doesn't matter how many ticks you get – unless all of them are ticks, in which case you know it's correct.

Male: You can ignore the ones of which the other side is 'both not true'?

Lecturer: Anywhere it's not the case that all the left-hand side are true, that's not of interest, because you haven't got a situation where the premises are all true.

An argument is only good if the premises are all true *and* it's valid, i.e. it can't be the case that the premises are true and the conclusion false.

Male: However complicated, one cross means the whole argument fails?

Lecturer: However complicated, one cross means the whole argument fails.

Male: I think earlier you mentioned 'affirming the consequent'?

Lecturer: The reason this is invalid is it's affirming the consequent, yes. This is a very typical fallacy. It's a very common fallacy. It is a fallacy because it's affirming the consequent, yes.

Male: We've only been dealing with 'P' and 'Q'. If you go [beyond the boundary and 'P', 'Q', 'R' and whatever, would it still work?

Lecturer: Oh, yes, it will work. It doesn't matter how many sentence letters you have, this will always work. It *always* works. You won't *ever* be left with uncertainty on this. You will always get an answer. Let's -

Male: Sorry, I've just realised what the fallacy is now you've explained it. ‘P therefore Q’, ‘P’ so ‘Q’, this is ‘if P then Q’, ‘Q’ then ‘P’ …

Lecturer: Yes, except you shouldn’t be interpreting that [the sequent] as ‘then’. This is not an ‘if/then’.

Male: No.

Lecturer: There’s a big difference between implication and entailment.

So there you are. (Slide 28 again) This is exactly what we’ve just done. In row one, the formulae to the left-hand side are all true, but so is that to the right, so that’s not a counterexample.

But in row three, the formulae to the left are all true, but the formula to the right is false, and that’s exactly what we need for a counterexample. That’s enough to tell us that the whole sequent is incorrect.

Male: So it doesn’t matter, in this context, if there is no situation in which both propositions are true? If you constructed a sequent and you built your table, and there were no examples of where all of the propositions were true on the left-hand side, that wouldn’t matter?

Lecturer: No.

Male: It doesn't matter that there are no ticks. It only matters if there's one cross.

Lecturer: The crosses are the only things that are really interesting. We only put the tick in because you see 'two trues'. All the premises to the left-hand side are true.

Male: If there are no ticks and no crosses, what then?

Male: It's still a valid argument…

Lecturer: It might be all like that. That's fine.

Male: Is it a valid argument?

Lecturer: It's not invalid. (Laughter) Given that it's either valid or invalid, it not being invalid means it's valid, yes.

Male: Is this the root of the reason why a lot of science is done by disproving stuff rather than proving stuff?

Lecturer: Yes.

Male: Because this process can't *prove*, but it can invalidate.

Lecturer: The reason that Popper was so anti-induction is because induction never gives us certainty. He thought that what we need to do is falsify arguments instead of proving them, because you can’t prove anything, but what you can do is falsify them. You can deductively falsify things.

Male: Yes, which is what we’ve just done, but that’s the basis of -

Lecturer: Popper’s …

Male: That’s the basis of the null hypothesis test, with clinical trials in epidemiology. You always prove that the null hypothesis is wrong. You never prove your hypothesis is right.

Lecturer: Yes. You shouldn’t be trying to confirm your thesis. You should be trying to ‘*dis*confirm’ it. You should be trying to falsify it.

Male: Yes.

Lecturer: (Slide 29 again) Good. So we now know that the sequent ‘P arrow Q’, ‘Q’ – you see, I'm doing it now – ‘sequent’ ‘P’ is incorrect, and the argument therefore invalid.

The interesting thing is we’ve also got the counterexample to the argument. We can say that this argument is invalid whenever ‘P’ is false and ‘Q’ is true.

If we had an interpretation, which we don't, because we've just been working with the sentence letters, but if we had an interpretation we can get exactly what the counterexample is. So we could see under what conditions this argument is invalid.

So not only do you know with complete certainty that it is invalid, you also know *when* it is invalid, what the situation is that makes it invalid.

Well, so far we've done baby steps, and these are really easy arguments. We knew before we even started whether they were invalid or not.

(Slide 30) Now let's do this one. So this is the sequent. Do you remember what that's about? Does anyone remember what it's about?

Female: Tickling cats.

Lecturer: Tickling cats, exactly so. That's the formalisation of the 'If you tickle her you deserve to get scratched' type of argument. If you remember, that was the interpretation. (Slide 31) It's all about whether she wanted you to tickle her, and whether you went about it in the right way, and so on.

To test this sequent, naturally we first draw the truth table. Okay, draw the truth table. (Slide 32)

Female: Are you telling us to?

Lecturer: Yes. (Laughter) Well, you know the algorithm … So you need to work out how many columns and how many rows.

Female: Can we have it up again, because I can’t remember it.

Lecturer: Yes, sorry. There’s the formula. Has anyone worked out how many columns we’ve got yet?

Male: Yes.

Lecturer: Put up your hand if you have. Well done. How many?

Female: Ten.

Lecturer: Ten? Is that what everybody has got?

Male: Nine.

Lecturer: Some of you think ten, some of you think nine. I think it’s nine, actually.

Male: Nine, yes.

Lecturer: You're getting an extra one from somewhere.

Male:	Yes, I'm going with ‘not P’ column, separate to the …
Lecturer:	No, that’s a mistake.
Male:	Oh.
Lecturer:	Well, where would you have a ‘not P’ column?
Male:	[inaudible discussion]
Lecturer:	Well, then you shouldn’t have that, should you?
Male:	Oh, okay.
Lecturer:	Yes. There are nine columns. Okay, good, nine columns. Now I want to know how many rows there are.
Male:	Sixteen.
Lecturer:	Sixteen? Well done. You’re getting this very fast. Well done.
Female:	I have to make this, I tell you now.

Lecturer: So here we are. There we are. (Slide 33) Nine columns and sixteen...

The only reason we work out sixteen is we've got 'true false, true true false false, true true true true false false false false, true true true true true true true true, false false false'...

You can see why truth tables are a bit of a bore, and why you will be very glad to get onto tableau tomorrow, which are much, much easier!

Tableau work just as well as truth tables, but they're much, much easier, but only after you've learnt how to do them. Truth tables …

Female: Can't we do those first?

Lecturer: No. (Laughter)

[inaudible discussion]

Lecturer: How many columns there are? Well, I gave you the formula earlier. If you've got two sentence letters there are four. If you've got three sentence letters there are eight. If you've got four sentence letters there are sixteen.

[inaudible discussion]

Male: It's two raised to the power of the number of letters.

Male: Yes: two squared by nine, four squared.

Lecturer: There you are.

Male: No, it's not squared, because you've got …

Male: No, sixteen is three squared by four squared. It's a nine to sixteen matrix, isn't it?

Lecturer: Ask him, not me. (Laughter)

Male: This is ....

Male: In the first column the convention to have all Ts and then Fs is obvious. In the second, third, and fourth column is there anything simple to remember the convention, or do you just …

Lecturer: What you've got to have is the situation for all the possibilities, and this is just a way of making sure you've got that. So this is 'true true true true, true true true, false, true true true, false, true true, false false', etc.

Male: So, if we were doing it on our own, certainly as long as we have all the combinations we'd have it, but this is the convention.

Lecturer: This is the way we are doing it, yes.

Male: Right. Because you could have all Ts in the second row …

Lecturer: Well, this is the way to remember it. It's knee-jerk stuff. It's dead easy to do this once you've started to do it.

Now we need to start doing the formulae. (Slide 34) Now, every one we've done so far, we've done complex formulae but we haven't done one with two truth-functors in. Here we've got one formula that has two truth-functors. Which one is it?

Female: The last one.

Lecturer: It's the last one. It's the one on the right-hand side. You've got a negation sign *and* a conditional.

What are we going to do first, do you think: the negation sign or the conditional?

Male: Well, the negation sign only applies to 'P' in this case.

Lecturer: Good, yes. We do the truth-functor with the smallest scope first when we're doing truth tables.

The one with the smallest scope, we've got the 'not-P'. The scope of the 'not' is just 'P'. If the 'not' were outside the brackets, the scope of the 'not' would be 'P arrow S', but here it's just 'P'.

So what we're going to do is, in lowercase letters, we want to put in the truth value of 'P'. And how are we going to do that?

Male: Just reverse the first P?

Lecturer: It's just going to reverse whatever we'd had in 'P'. So it's the 'P' that's negated, so this is going to reverse whichever truth value 'P' has in the key. (Slide 35) So there we are. 'P' is true, 'not-P' is false. 'P' is false, 'not-P' is true, and so on. Do you see how that works?

So having done the 'not-P', we now want to put in the conditional. This is where I warned you before: you've got to be very careful about using the truth table definition of the conditional, because this is the *antecedent*: not 'P'. Are you with me?

Male: Yes.

Lecturer: So here we've got 'S'. (Slide 36) Well, we could actually put in the truth – I think I've done that, actually. Let me see whether I have. Yes, I have. I've put in the 'S' here. (Slide 37)

You will notice that this is exactly what's there has gone in here. That's just to make it much easier for us to fill in the truth value of the whole formula here, because we've actually go the truth values on each side.

As you get more practiced you don't have to do this. You can just do it from the key. But let's not do that. Let's not run before we can walk. So what do we put: false, true? What's the truth value for the conditional? (Slide 39)

Chris: Whatever it is.

Lecturer: True or false? 'Whatever it is', says Chris. (Laughter)

Male: True.

Lecturer: That's true. Okay, what's 'false, false'?

Male: True.

Lecturer: 'False, true'?

Male: True.

Lecturer: Actually, you will notice that every time the antecedent of a conditional is false, the conditional is...?

Male: True.

Lecturer: (Slide 39) True. So we can put ‘true’ in all of those. Are you with me?

If you look at the truth table definition for the conditional, you will see that whenever the antecedent is false the conditional is true. So, as the antecedent is false in all these, all of these will be true. So actually we can just put ‘T’ down all those and start looking here.

When the antecedent is true, and the conditional is false, the truth value is...?

Male: True.

Lecturer: True. When the antecedent is true and the conclusion is false, false. So we’ve got our first ‘false’ here. True or false?

Male: True.

Lecturer: True. True or false?

Male: False.

Lecturer: False.

Male: True.

Lecturer: True, false, true, false. Are you with me?

Male: Yes.

Lecturer: So there we go. Have a look at that and see if there's anything you don't understand. Put your hands up if there is.

Next we're going to do this one. Looking at your truth table definition for 'or', see how to fill those in.

That one's quite easy, because you've got 'PQ' in the right order there, so actually that's quite a nice one to do.

Let's move on. There's one. (Slide 40) That's the truth table definition for disjunction – the 'or' – and that tells us that it's only false when 'P' is false and 'Q' is false.

So when we fill that in here, (Slide 41) if you look at 'P' and 'Q' here, we're looking for when they're both false, and actually that doesn't happen until down here, and there are four rows in which they are both false, and in each case each of those is false, otherwise they're true. Are you with me?

Female: Yes.

Lecturer: So 'P' or 'Q' is true, then 'P' and 'Q 'are both true. When 'P' is true and 'Q' is false, 'P or Q' is true. The only time where 'P or Q' is false is where 'P' is false *and* 'Q' is false. Are you with me?

Male: Yes.

Lecturer: So that one's filled in. Again, it just comes mechanically from the truth table definition of 'or'.

Now we need to do the two conditionals. (Slide 42) The hardest thing about these is getting the – in each case they are in the same order – but it's getting the truth values in the right place from the key.

Okay, 'Q arrow R'. Tell me why that's true, somebody. Anne, can you tell me – sorry, picking out Anne is so mean, but go on. Can you tell me why that's true?

Anne: Because 'S' is true.

Male: Which one is this?

Lecturer: That's not true, because 'S' is true. That's not the only reason.

Male: It is.

Lecturer: Well, yes, but it's not from what I'm … because 'R' is false and 'S' is true. So that gives you true here, because that's what, again, the truth table definition says.

Why is that false? Can somebody tell me that?

Male: ‘P’ is false.

Lecturer: Because ‘R’ is true and ‘S’ is false, and therefore ‘if R then S’ is false. That’s the only time when ‘if R then S’ is false, is when the antecedent is true and the consequent is false.

(Slide 43) Does everyone see where these are coming from? Or if they don’t see particular ones, they’ve got the general idea?

Male: You say, ‘Do you understand it?’ I say, ‘I can fill the table in, but I'm not sure I actually understand what it means.’

Lecturer: Being able to fill the table in is quite enough at the moment. Well, you *do* understand what it means, because what you're doing, we know that each one of these truth-functors is truth functional. So the truth value of the whole depends only upon the value of its parts.

Male: Yes, I understand that.

Lecturer: All we’re doing is saying what the impact of the combination of the truth value of the parts is.

Male: Yes.

Lecturer: We know how to work that out from the truth table definitions.

(Slide 44) Now we can fill in this column, which will tell us whether the argument is valid or not, or whether the argument sequent is correct. So is there a possible situation there in which *all* the premises are true?

Female: Yes.

Lecturer: So give me the numbers of the rows. So number one, row one is true true true. Good. Two is true false, so it doesn't count. So the next one is this one, isn't it? True true true. This one is true true true. This one and this one … So quite a few of them, actually.

Male: There are five.

Lecturer: Here they are. (Slide 45) There we are. There are lots of them in which the – well, I've filled in the tick already, so no.

We now need to check each of these to see whether we've got a false on this side, and wherever all the premises are true the conclusion is also true, isn't it?

Male: Yes.

Lecturer: So there isn't a counterexample. There is no possible situation in which the formulae on the left-hand side are all true and the formulae on the right-hand side are false. Therefore the sequent is correct. (Slide 46)

Male: Would I be safe in only considering the ones that are false on the right-hand side and not being interested in the rest? Because to find the counterexample …

Lecturer: Well, that's not a counterexample, is it?

Male: No, but I'm only going to look at the ones that say F on the right-hand side.

Lecturer: Yes.

Male: I don't need to discuss any of the ones that say 'T'.

Lecturer: Well, it doesn't matter which way round you go. You have to look at whether what's on the left-hand side is true and what on the right-hand side is false. You can start by looking at what's false on the right-hand side.

Male: Just that there are fewer of them …

Lecturer: Yes, it doesn't matter at all.

Male: There are only four places where the sequent comes out as an F.

Lecturer: One, two, three, four.

Male: So you only need to look at those rows.

Lecturer: You could look there.

Lecturer: Yes.

Male: But they've all got an 'F' in them, so you can...

Male: So there's no counterexample?

Lecturer: Yes, there's no counterexample. So that argument claim is correct. There is no possible situation in which the formulae to the left-hand side are true and the formulae to the right-hand side are false. Therefore the sequent is correct, and we know that the argument is...?

Male: Valid.

Lecturer: Valid, yes. So the sequent is correct, the argument is valid. (Slide 47) This is the argument with which we started, and we know for absolutely certain, *completely* conclusive, that this argument is valid.

There is no possible situation in which the premises of that argument are true and the conclusion of that argument is false. So it's a good argument. (Slide 48) There are some semantic sequents for you to practice on.

Male: Thank goodness! (Laughter)

Lecturer: And incidentally, what do you think this means?

Male: Always true?

Lecturer: There is no possible situation in which what's on the left-hand side is true, is all true.

So what that's telling you is that that (the formula) is inconsistent. There is no possible situation in which the formulae on the left-hand side are all true. So in other words that must be inconsistent. There is no formula on the right-hand side, so you can ignore it.

This one tells us what? That says that that's (the formula) a contradiction, so in every possible world that's false. That tells us what?

Male: It's always true.

Lecturer: That this is a necessary truth. So there’s no possible situation in which the formula on the right-hand side is false. That’s always true, in other words. So you can read it in exactly the same way. You just haven’t got one of the sides to worry about.

Male: It says answers in your answer booklet on page seven. Are we still waiting for the answer booklets?

Lecturer: You are, yes. I've got the answer booklets here. I will give them out. You didn’t think I was going to give them to you. (Laughter)

I'm a bit unsure about giving them to you tonight, because you might look up tomorrow’s answers. The only thing is that I'm absolutely confident that, if you do look up tomorrow’s answers tonight, they won’t mean anything to you. (Laughter)

Okay. We’ve actually got five minutes for questions.

Male: Can I just ask a question?

Lecturer: Yes.

Male: I'm very bothered about the rows that you just put dashes in, how they don’t seem to contribute at all to the validity, or otherwise, of the argument.

Lecturer: There are two questions to ask of an argument. Are all the premises true? Is the argument valid?

Male: Yes.

Lecturer: If it's not the case that all the premises are true, then actually you're not interested.

Male: Oh, okay, yes.

Lecturer: Let me be careful about that. In a truth table you've got all the possible combinations, and that's the important thing.

Male: Of 'P', 'Q' and 'R', or whatever your …

Lecturer: Or of whatever your premises are.

Male: But then you've got to have all the arguments true?

Lecturer: Not arguments. All the formulae on the left-hand side, i.e. all the premises, have got to be true, before we are interested in the truth-value of the conclusion.

Male: If they're not true that contributes towards a dash?

Lecturer: Yes. Well, that *is* a dash.

Male: That is a dash?

Lecturer: Yes. So if some of the premises are true, some are false [cross-talking] …

Male: Well, then you can't tell anything about the argument, basically?

Lecturer: Yes.

Male: Okay.

Female: On page four, the bit at the bottom, where it talks about 'P' being false and 'Q' being true.

Lecturer: Page four?

Female: Yes. But then 'if P then Q' is true. Is there somewhere I can get more information on how that's logically derived? Because I kind of struggle with that intuitively.

Lecturer: Looking at four.

Female: Page four. It’s the definitions, I think.

Male: It’s what we discussed earlier.

Male: The bottom of page four.

Male: ‘If P then Q’ where ‘P’ is false and ‘Q’ is true.

Lecturer: Are you talking about slide fifteen?

Female: Slide eight.

Lecturer: Eight?

Female: I think it’s slide eight.

Lecturer: So it’s not page four.

Male: We’ve got two per side.

Lecturer: Oh, yes, I'm sorry. Give me one with two per side.

Male: The slide numbers …

Lecturer: Oh, right, the definitions. Sorry, go on. What was the question again?

Female: I'm just struggling a bit with ‘P’ being false and ‘Q’ being true, and then ‘if P then Q’ being true as a result. So I was just wondering if there’s anywhere I can look up more about the logic behind that.

Lecturer: That’s the one that Kirsten is going to ask me about in the question and answer time tomorrow. Well, we could do it very quickly now.

Well, no, I think it’s probably better to do it tomorrow in the morning. Because that is very difficult to motivate, and I can quite see why you're having trouble with it. I will try and motivate it for you tomorrow.

Female: Thank you.

Male: A few moments ago, when you mentioned Popper, in your example previously, where there’s no counterexample, in his language that would be not falsifying it?

Lecturer: No, it’s ‘not falsified’.

Male: Not falsified?

Lecturer: Yes. Think of it like this. Think of the logic of investigation, if you like.

You start here with sightings of white swans. By *inductive* logic you start to say, ‘All swans are white.’ Every swan I've ever seen has been white, therefore by an induction, possibly a rather bold one, I form a hypothesis that all swans are white.

Then I test that hypothesis by saying, ‘well, if all swans are white, then anything that *is* a swan will be white’. Well, of course that’s not going to be true. If I see a black swan, that’s falsified that immediately. That’s by *deductive* logic. So deduction falsifies.

So every white swan I see is not very interesting, because it’s just yet another little confirmation of my hypothesis, for which I already had – every swan I've ever seen has been white. ‘Oh, look, there’s another one, and another one, and another. Yawn. (Laughter) There’s a green one. Now we’re talking.’

This is really interesting, because once you’ve got a green one you know that that (the hypothesis) is wrong.

Female: You only need one of them.

Lecturer: You only need one of those and you’ve completely conclusively falsified that.

Male: By knocking out the false one …?

Lecturer: Actually, you haven’t quite, because the other thing. There’s always some interpretation. Because of course you might see a black swan going up the Swan River, in Perth, and you think, ‘that can’t be a swan. It’s not white.’ (Laughter)

Male: You could look at it.

Lecturer: Yes. ‘It looks like a swan, but it’s obviously not one, because it’s not white, and if all swans are white...’

Another one that goes like that is, ‘All women are passive. Mrs. Thatcher isn’t passive. Therefore Mrs. Thatcher ... Well, either you will falsify your premise and say, ‘It’s not the case that all women are passive’, or, ‘Mrs. Thatcher isn’t a woman.’ That’s where the joke ‘Mrs. Thatcher is the best man in the Cabinet’ [came from]. (Laughter)

Male: False negatives or false positives. Because a black swan that you see might be a very dirty white swan!

Lecturer: Well, but then it’s the premise isn’t true, is it?

Male: Well, you think you’ve found the exception that allows you to dispense with the inductive rule that all swans are white.

Lecturer: No, that's an – sorry, inductive, you said that, yes.

Male: You think you've found the overturn of that rule, but in fact you have made an error.

Lecturer: But remember that the deduction tells you that *if* the premises are true the conclusion must be true. What you're describing is a situation where one of the premises *isn't* true. It's not the case that this swan is black. It's a white swan that's dirty.

Male: But you do get false positives and false negatives …

Lecturer: Well, yes, but this doesn't change the fact that if the premises are true the conclusion must be true, and that's what deduction gives you.

On one side you've got induction. You form your hypothesis on the basis of inductive evidence. Then you use deductive logic to test ...

Male: To disprove it?

Female: Is there some way of 'weighting', where if it's inductive, if you have seen 50 swans …

Lecturer: Oh, you have to.

Female: … And you have another person who has seen 60 swans that – do you know what I mean? It progresses. It becomes a statement that is now worth 60 swans.

Lecturer: Well, the reason that science is a collaborative enterprise is that you're always wanting people to confirm your – you don't know whether it's just happening in your lab, whether in Australia things are completely different, as indeed they were with swans.

That's why you want other people to replicate whatever it is you've concluded, because each replication is further confirmation.

But of course it doesn't matter how often you confirm the thing, you've never got certainty, because you never get certainty with an inductive argument.

Female: You get increasing …

Lecturer: … you falsify. Well, actually, remind me. I will talk to you, if you like, if you ask me tomorrow, which would be very nice of you, about the paradoxes of confirmation. I will talk to you about black ravens, and white gym shoes, and things like that, and about grue. That will be fun …

Male: This is why scientists eventually stop doing experiments. You see one white swan …

Male: No.

Male: No, the way you do an experiment is you – the way you do epidemiology, in clinical trials, is you turn that on its head, and then seek to...

Lecturer: Well, you do exactly what I...

Male: The epidemiologist would not phrase the question that way. I think they would phrase the question: 'There are some swans that are not white.' Then they would look at evidence to discard that hypothesis. They ask the question a very odd way. It takes a long time to get your head around the way they...

I may be mistranslating this. If you set out to say, 'My treatment is better than the gold standard of care', that's not the question you ask. That's what you want, but that's not the question you ask. The question you ask is, 'Can I find evidence to reject the idea that my treatment is *equivalent* to the gold standard?'

Lecturer: Yes, but that's exactly what I'm saying.

Male: Which is how they set out to do their experiments.

Lecturer: That's exactly trying to falsify your hypothesis, which is that your treatment …

Male: Is the same as.

Lecturer: Either the same as, or better, or whatever …

Male: They always set out to falsify the idea that the two are equivalent.

Female: So they're postulating?

Male: If I'm getting this right, they’re falsifying the opposite of your hypothesis. Is that what you're saying?

Male: Yes, I think so. I have a bright idea that I can cure cancer, and the gold standard of care is radiotherapy. So I'm claiming that my idea is better than radiotherapy.

The way I would phrase the clinical trial is, ‘Radiotherapy is equivalent to my bright, new idea.’ So I would phrase the hypothesis, ‘My good idea is the same as radiotherapy’, and then I would look for statistical evidence to reject that idea, or fail to reject that idea.

Lecturer: So you would design an experiment or a trial which would show that P is not equivalent. Actually, you don’t want equivalent. Radiotherapy is worse than P.

Male: I want to reject the idea the two are equivalent.

Male: But you design it …

Lecturer: So you can either show that P is better or P is worse? Would either …

Male: Well, yes, there are statistical tests. There are two different types of statistical test: one of which will show you which way, and the other which just says, ‘They're not equivalent.’

Lecturer: Sadly, as nobody ever publishes negative results, we will never actually know whether you succeeded in showing this.

Male: That depends whether you work for the university or [inaudible]. (Laughter)

Male: A physicist would publish the negative results. That’s all they're interested in doing is publishing …

Lecturer: Well, they’d get it put in a journal.

Male: When they discovered that neutrinos went faster than the speed of light, that got published, whereas ...

Lecturer: Well, yes, but...

Male: It was completely wrong, but when they were promoting …

Lecturer: And took a long, long time before they agreed that they had published it and they probably shouldn't have done.

Male: Oh, yes. That's also true …

Lecturer: Well, they wanted to get everybody working on what they'd done wrong.

Male: Well, they just wanted to know what they had done wrong. They knew they had got something wrong, but they had no idea what it was.

Lecturer: Let's go, because it's well after time, and I need to go home. (Slide 49).