A Romp through the Foothills of Logic: Session 2

You might find it easier to understand this podcast if you first watch the short podcast 'Introducing Truth Tables'.

Lecturer: (Slide 2) Right, by the time we finish this session – ha ha – okay, [inaudible] note here, bit rash – you're going to have the tools that you need – notice that I'm not saying that you will be able to – you will have the tools that you need to translate and analyse English arguments into sequents i.e. argument-claims of propositional logic.

Now, normally I'd say, 'in the propositional calculus', but I was convinced by somebody that is less intimidating to say, 'propositional logic' than to say 'the propositional calculus'. There's nodding, so there's at least one person who agrees with that.

Okay, so you'll certainly know what a sequent is, and you'll know what propositional logic is, but... Okay, so you'll have the tools you need to translate an argument that's been set out in logic-book style, that's an analysed argument, into a sequence of the propositional calculus.

(Slide 3) So, you've already learned how to analyse arguments, i.e. you've learned how to set them out logic-book style, and this is hugely important because trying to formalise an unanalysed argument is so stupid – it really... you'd be drummed out of OUDCE trying. Notice it is silly because before you've analysed an argument you haven't got rid of the irrelevancies, you haven't added the suppressed premise. You've still got inconsistent terms. All sorts of things you want to get rid of before you start formalising.

Because formalising is actually the hardest thing you're ever going to be able to do – or you’re ever going to learn to do. Once you've formalised, evaluating an argument, believe me, is quite easy. That's the thing you're going to impress people with when you go home tomorrow afternoon, you're going to be able to use symbols in such a way that everyone's going to say, ‘ooh.’

But actually, the really hard thing is what we're doing in this session. All I can do in this session is give you the headlines and the handout I've given you, an expansion on the headline stuff and the other podcasts will help you on those as well. We have to learn to formalise something before we can apply the rules.

(Slide 4) Okay, a word about terminology: I've been using ‘sentences’, ‘statements’, and ‘propositions’ pretty well interchangeably, which is a bit naughty of me, really. *Sentences* can be used to make *statements*. They're sometimes not used to make statements, I mean, if I write, ‘James is tall’ on the board here, I'm not using it to make a statement. I'm just using it as an example. I'm *mentioning* this sentence. I'm not *using* it. So it's a *sentence*, but it's not a *statement*, because it's not expressing any belief that I have, or that anyone else in this room has. There's no one I'm intending to refer to by using 'James' and so on.

So a statement is *a sentence in use*. A proposition is the *content* of a belief or a statement. So if I expressed the belief 'James is tall', then I have a belief with the content [James is tall] and I'm expressing that belief by means of a statement with the meaning, or the content, 'James is tall'.

So that's where the idea of *propositional* logic comes from. You don't get propositional logic if you don't have contents of beliefs and meanings of statements. You'll see this becomes important later on, but you don't need to worry too much about it now.

(Slide 5) The first thing you've got to learn in this session, really important: you need to distinguish those arguments that you can formalise from those that you can't formalise. Sorry, that you can formalise *in this particular language* that you're learning.

Now, what you're learning is what is referred to by people disparagingly as 'baby logic', but you have to start early on. You cannot run before you can walk, etc., etc. You are going to be learning the language of propositional logic; that leaves an awful lot of arguments that you won't be able to formalise.

So I need to tell you what those arguments are just so that you don't think that you can go tomorrow morning or Monday morning to your newspaper, take the leader, and immediately formalise it in the language of the propositional calculus, because that's hugely unlikely that you’ll be able to do that, and this is why.

(Slide 6) Propositional logic cannot be used to test inductive arguments. Well, immediately that's a whole slew of arguments that you're not going to be able to use the method I'm showing you today to formalise.

It can't be used to test deductive arguments whose validity depends on anything other than their sentential or propositional structure, and I'll explain what I mean by that in a minute.

It also can't be used to formalise any deductive argument that depends on non-truth-functional sentence connectives. Now, you always suspected that, but I'm just telling you that that is the case, okay?

(Slide 7) So you can't do inductive arguments at all. It's only deductive arguments you can do, and of the deductive arguments that you'll come across you can only do ones that are not of these kinds.

So let’s have a look at these. That really is a major problem. I mean, don't think – inductive logic is miles behind deductive logic in our ability to formalise it, and you can probably tell me already the reason for this. Can anyone tell me already the reason for this, given that you know what distinguishes deductive arguments from inductive arguments?

Male: You have to look at the external world in order to…[inaudible].

Lecturer: You cannot evaluate an inductive argument without bringing in loads of background knowledge about the external world – well, forget about the *external* world, about the world. For making sense of that, systematising that is a nightmare. I mean, we do quite a lot with the probability calculus and things like that, but we're miles behind deductive logic. Deductive logic we can formalise really very, very well. But there are these two types of argument that you can't do.

I've just said all that. So the formalising of an inductive argument is just way behind, so you're not going to be able to do any inductive arguments as a result of this weekend.

(Slide 8) But there are two categories of deductive argument for which propositional logic is no use at all, and that's these two categories. The two that I've already mentioned. (Slide 9) So, what is an argument, the validity of which turns on something other than its propositional or sentential structure?

This is why I gave you the three ‘sentences’, ‘statements’, and ‘propositions’.

(Slide 10) Well, okay, here are two arguments. ‘If it's Friday, then Marianne is wearing jeans. It is Friday; Marianne is wearing jeans’.

‘Marianne is wearing jeans. Marianne is the Director of Studies in Philosophy at OUDCE’. Every title I have is so long. Conclusion: ‘The Director of Studies in Philosophy at OUDCE is wearing jeans’.

Now, which of those is such that its validity turns entirely upon its propositional structure, and which depends on something other than its propositional structure? Put up your hand when you think you know.

Okay, some.

Female: I think it's the second one, because [Cross talking] whatever where Marianne is not necessarily the Director of Studies.

Lecturer: So you think that one depends on something other than its propositional structure? Is that what everyone else thinks?

Male: Yes.

Male: No, they're the same.

Lecturer: Erm...

Male: In form, not in content.

Lecturer: Right, they're not the same argument. They're very different arguments. They're both about me.

Male: They're both about you and they're both about wearing jeans. They're both about Friday.

Lecturer: Friday doesn't appear in that one.

Male: Oh.

Lecturer: Okay. So, what was your...

Female: It doesn't involve Friday.

Male: Well, [inaudible].

Lecturer: I don't think it's because it doesn't involve Friday that that's not dependant on its-

Female: I guess it's – my initial thought was that both premises are about you, and there's a world where you're not the Director, and where the Director of Studies could be wearing jeans, and that's just not you.

Lecturer: But it's not about – yes, okay. Good. Whereas this one has- this one is valid – do you want to say something quickly?

Male: Can you do the first one in terms of, ‘If P then Q’?

Male: Yes, precisely

Lecturer: Yes, and that one you can't.

Male: And that one you can't.

Lecturer: Good. You're absolutely right. But actually you're saying the same thing; you're giving the reason why that is the case. This one, you could say, ‘If P, then Q, P, therefore Q’, can't you, with the interpretation, ‘P: it is Friday. Q: Marianne is wearing jeans.’

This one, you can't do that because this one goes *inside* the sentence, doesn't it? The validity of this argument (the RHS) depends upon the fact that you've got a subject-predicate sentence, so *the Director of Studies in Philosophy at OUDCE*, that's the subject, *is wearing jeans*. That's the predicate. Do you see what I mean?

So all of that is a proposition. All of that is a proposition, and that is a proposition. We've got to look i*nside* the proposition to get the validity of the argument, haven't we?

(Slide 11) If you were going to – actually, we're doing this later, but we may as well do it now – if you were going to put this into ‘P’s and ‘Q’s, this would be ‘P, Q, therefore R’. Where's the argument? It's gone.

Whereas this one, you've got complex sentences: ‘If P, then Q, P, therefore Q’. Do you see the difference?

This is not something that – if you're feeling unsure about that, don't worry. It's something that is reasonable to be unsure about at this stage in the game.

Female: Does that pivot round the 'if, then'?

Lecturer: That pivots round the 'if, then'? That's right. 'If, then' is a *sentence* connective. 'If, then' takes sentences rather than predicates.

Female: There was no 'if', there's no conditional.

Lecturer: There's no sentential connective in here. There's no sentence connective. There's no 'therefore', there's no truth-functional sentence connective. But we'll get onto that in a bit-

Female: Could it be 'P and Q' 0:11:49] and a conclusion P and Q [inaudible]?

Lecturer: How could that be ‘P and Q’? What's the conjunction here?

Female: Well, I just thought that Marianne is found in both of them, so it could go [Cross talking].

Lecturer: I'm not saying this can't be formalised. I'm saying it can't be formalised in propositional logic. The way this would be formalised is: 'F of A' – so A is Marianne and F is wearing jeans – 'A is B, F of B'. Do you see what I mean? So B would be the – so this can be formalised, but it can't be formalised in propositional logic. We lose the argument if we try and do it. It's just too coarse-grained to-

Male: So the one on the right is the centennial structure example.

Lecturer: No. The one on the left is – that one has-

Male: Sorry – the non-centennial structure-

Lecturer: Sentential.

Male: Sentential.

Lecturer: Or 'propositional'. We have to look inside the sentence. We have to look inside the proposition in order to see the validity of that one, but that one, we don't.

Female: Is that still 'valid', as it were? The one on the right [inaudible].

Lecturer: Yes. Both of them are valid, but that's valid by virtue of its sub-sentential form, not its sentential form.

Female: It's valid because it happens to be true?

Male: No.

Female: No.

Male: No.

Lecturer: Somebody hit her. No, validity and truth are not the same thing. Validity is defined in terms of possible situations. Truth is defined in something I'm not going to talk about now. You can listen to the podcast on truth if you want to.

Male: Is propositional logic the same as first-order logic?

Lecturer: Erm, no, because – yes, okay. First-order propositional logic, but you can also have first-order predicate logic.

Male: Is this predicate logic, you require [inaudible]

Lecturer: This is predicate logic (the RHS argument on slide 10). Yes. So this is propositional logic. You need propositional logic to do predicate logic, but you don't need predicate logic to do propositional logic, which is why propositional logic is – well, actually predicate logic's 'baby logic' too, I hate to tell you. You are only learning propositional logic.

Male: Predicate logic is the one with the upside-down ‘A’s and backward ‘E’s and all those things.

Lecturer: That's the one. Just forget about all that. We won't be doing that. We've only got a weekend, for God's sake!

Okay, so do you see why one of those can be done in propositional logic, the other can't? And if you don't see why, there's stuff in the handout that will help you. Later on, we'll come back to this. We'll try formalising the second argument, see what happens.

One argument that you're not going to be able to formalise is any argument that depends for its validity on its sub-sentential structure. The second argument that you're not going to be able to formalise is any argument that depends on a non-truth-functional connective.

You see, it's going to be lovely to go home and show off your understanding of this...

(Slide 12) Okay, what's a non-truth-functional connective? (Slide 13) A truth-functional connective is a species of sentence connective, and a sentence connective connects sentences. We love this technical terminology, don't we?

Okay, a sentence connective takes one or more sentences to make another sentence. (Slide 14) So, for example, English has got lots and lots of sentence connectives. Here's one: 'P and Q'. The conjunction sign 'and' is a sentence connective, isn't it? It can also connect predicates. ‘the black and white cat’, 'and' is connecting two predicates there. So you can't formalise 'black and white cat' in terms of propositional logic, but, ‘It's raining and it's Friday’, the 'and' is connecting two sentences. So 'and' functions as a sentence connective.

‘It's probably the case that P’ – now, you might think it's odd calling that a ‘*connective’* because it's only taking one sentence, but we call that a 'unary' connective rather than a 'binary' connective. So, ‘it's probably the case that P’ takes a sentence, ‘P’, to make another sentence. So: ‘It's half past five, it's probably still light.’ Or: ‘it's still light, it's probably still light.’ Those are two different sentences, and what makes it different is you're operating on the – we sometimes call connectives 'operators', by the way – you're operating on the sentence 'P' with the connective, ‘It's probably the case that’

Male: Why doesn't 'probably' make it an inductive argument?

Lecturer: I explain that in the book. Can I refer you to the book? I'm not going to explain that now; it'd take too long. Sorry.

‘P or Q’ – can you see that 'or' is a sentence connective? It takes two sentences, 'P' and 'Q', and makes another sentence, ‘P or Q.’ ‘It's Friday or it's October.’ Sorry, bad example, but I hope you can see what I mean.

‘P because Q’ – another sentence connective. So, P occurs because Q occurs.

‘It's not the case that P’ – another unary connective. So we take the sentence 'P' and operate on it with the negation connective to make another sentence, i.e., ‘Not-P’, or, ‘It's not the case that P.’

‘It's necessarily that case that P’ takes, again, 'P', and operates on it to give you another sentence.

So English is just full of sentence connectives. So is any other language. Any other human natural language. Each of these words or phrases – and there are many, many more – takes one or more sentences and uses them to make another sentence, a more complex sentence. We build sentences by means of these words, and connecting words.

Any questions about that? I mean, that's pretty straightforward. It gets harder from now on.

(Slide 15) So, as propositional logic works only on arguments whose validity turns on truth-functional connectives – and you've got to learn to recognise when a sentence connective is truth-functional and when it isn't – okay? You get that? So there's a class of sentence connectives, or 'truth-functors', or 'sentence operators', and then within that class there are the truth-functional kind and there are the non-truth-functional kind.

You can only formalise arguments that depend upon truth-functional connectives. Sorry, but that is the case.

(Slide 16) A sentence connective is truth-functional when, and only when – that's a variation on 'if and only if', by the way – the truth value of any sentence built from it is a function solely of the truth values of its constituent sentences.

Don't worry; it's going to get simpler.

A sentence connective is truth-functional when and only when – so a necessary and sufficient condition of a sentence connective being truth-functional – that the truth value of any sentence built from it, the complex sentence that you get from it, is a functional solely of the truth value of its constituent sentences. Let's have an example of that.

(Slide 17) Okay, this is another way of putting it: a sentence connective is truth-functional when and only when it has a complete truth table. Those are two equivalent definitions, and I'll show you what each of them means.

(Slide 18) The sentence connective 'and' is truth-functional, because if you take any two English sentences and connect them by means of 'and' the truth value of the resulting sentence is a function *solely* of the truth values of the two sentences connected. That means there's no need to know the *meaning* of those sentences to determine the truth value of the whole.

Let's see what I mean. Let's do a truth table, you're all familiar with truth tables.

(Slide 19) This is a truth table by which I'm going to show you the meaning. Now, do you remember that I said earlier that when you have a truth table, each row of the truth table is a different possible situation? Okay. So you've got row one, row two, row three, and row four. In row four, taking these different rows as different possible worlds, if you like, or different possible situations if you prefer.

Row one is the possible world in which ‘P’ is true and ‘Q’ is true. Row three is the possible world in which ‘P’ is false and ‘Q’ is true. So all I've done there is listed the different possible combinations of truth value. Do you see what I mean? One thing you might want to ask is, ‘Am I assuming that each sentence letter, or each sentence, can be either true or false?’ The answer is, ‘Yes I am’.

One of the rock bottom assumptions of classical logic, which is what you're learning, is that sentences have only one of two truth values. There is no third truth value, and then no truth value gaps. Neither of those things is obviously true, and so if I say that, ‘Mary's shirt is not *not*-red’ – actually, I'm wrong, aren't I – but anyway, it's not *not*-red... I mean, that doesn't mean it is red, does it? That suggests that there's a third truth value, there's something in between there, that it's neither true that it's – sorry. It's neither true it's red or false that it's red, because if it can be not *not*-red then it seems there's a third possibility, doesn't it? But we're ignoring that. We're doing classical logic and therefore we're assuming that every sentence has one of two truth values, and therefore that this is a representation of all the possibilities.

So ‘Q’ can be either true or false, and ‘P’ can be either true or false, and what we've done here is combined all the different truth values. Okay? Do you understand that that's a tabular representation of all the possible combinations of truth values of these two sentences?

Well, tell me: if ‘P’ is true and ‘Q’ is – so, forget these worlds for a moment. We're looking only at world one – if ‘P’ is true and ‘Q’ is true, what's the truth value of 'P and Q'?

Female: True.

Male: True.

Male: True.

Lecturer: See, you can do logic straight way. Okay: if ‘P’ is true and ‘Q’ is false, what's the truth value of 'P and Q'?

Male: False.

Female: False.

Lecturer: If ‘P’ is false and ‘Q’ is true, what's the truth value of 'P and Q'?

Male: False.

Female: False.

Male: False.

Lecturer: False. If ‘P’ is false and ‘Q’ is false, what's the truth value of-

Female: False.

Male: False.

Lecturer: False. (Slide 20) So, without knowing the meaning of ‘P’ *or* ‘Q’, you have managed to give me the truth value of 'P and Q' in every possible world, haven't you? That means, actually, that you know the meaning of 'and'.

I mean, that's very useful for you, isn't it? Here you are, you use this word all the time. (Slide 21) But this shows that you know the meaning of 'and' because what you know is the import of combining two sentences by means of 'and'. You know the ramifications for the truth conditions of 'and'.

If you try and teach a child the word 'cat', you try and teach the meaning of the word 'cat', you point to lots of different cats. Fat cats, thin cats, black cats, tabby cats, cats with a tail and cats without and so on. You say to the child at some point, ‘Is that a pussy cat?’ and the child goes, ‘Yes’ and you point to a dog and you say, ‘Is that a pussy cat?’ and then the child (you hope) goes, ‘No’.

What the child's doing is manifesting the fact that it knows the truth conditions, the conditions under which, ‘It's a cat’ is both true and false. So to grasp a set of truth conditions is to be able to determine the truth value of a sentence and to grasp the truth conditions of 'and' is to grasp the meaning of the word 'and'. You've just shown you understand the word 'and'. Well done.

Do you see that 'and' is truth-functional because the function of combining two sentences by means of 'and' – it's truth-functional because if you take two English sentences and connect them by means of 'and', the truth value of the resulting complex sentence is a function solely of the truth value of the two constituent sentences. You don't need to know what ‘P’ means or what ‘Q’ means in order to determine the meaning of 'P and Q'. Do you accept that?

Male: Yes.

Male: Yes.

Lecturer: Okay. Any more questions there? Okay.

Oh. First truth table and I've just drawn that. Okay, the truth table for 'and' can be thought of as the truth table definition of 'and'. We can define all the logical words used in the propositional calculus by means of these truth tables, and as I said, it gives us the truth value of 'and' in every possible world, so just as the child grasping the meaning of 'cat' enables it to determine the truth value of ‘that's a cat’ for any use of ‘that's a cat’ – it has the meaning. It knows the meaning of the word. It grasps the truth conditions.

So to be able to determine the truth value of a sentence constructed by 'and' in every possible world is arguably to know the meaning of 'and'.

(Slide 22) So let's see if the sentence connective 'because' is truth-functional in the same way. (Slide 23) We use the same truth table because it's a binary connective and we've already got that so we put that in there... okay, forgetting all these other worlds, looking only at world one in which ‘P’ is true and ‘Q’ is true, do we know the truth value of 'P because Q'?

Male: No.

Lecturer: Who thinks 'no'? You're absolutely right. We can immediately put a question mark in there, and we immediately know that this isn't a truth-functor. This is not a truth-functional connective. It may be a sentence connective, but it's not a truth-functional connective because its truth value depends – you have to know what ‘P’ means and what ‘Q’ means to know whether 'P because Q' is true.

If I put in that, ‘P: it's raining, *because* the precipitation...’ I can't finish that sentence, but I'm sure you can see where I'm going with that, then you might put a 'true' in there, but if I put, ‘Marianne's wearing jeans *because* it's Friday’ you'd put a 'false' in there, so it's a different truth value depending on the different meaning. Therefore, you can't determine the truth value of the whole on nothing more than the truth value of the parts. It's not a...

(Slide 24) So you can actually complete the rest of the truth... In the world in which ‘P’ is true and ‘Q’ is false, is 'P because Q' true or false?

Male: Could you say, ‘Because it is raining, the predicted precipitation... is a truth value of one’ true? In other words, the weatherman was right.

Lecturer: Well, we're not looking at the meaning of ‘P’ or ‘Q’. We're trying to work out whether we can determine the truth value of 'P because Q' in the world where ‘P’ is true and ‘Q’ is false.

Male: Oh.

Lecturer: So if ‘P’ is true and ‘Q’ is false, is 'P because Q' true or false? Or do we not know?

Male: [inaudible].

Female: It's got to be false.

Male: False.

Lecturer: It's got to be false.

Male: But it can be true [inaudible].

Lecturer: So we can do a partial truth table for 'because'. Well, if ‘Q's not true, then how can P be true *because* of Q?

Male: Because that's the relationship between them.

Female: Yes.

Male: Maybe P is true, [Cross talking] *because* Q is false.

Lecturer: Oh, I see what you mean.

Male: I don't think [Cross talking] for any of those.

Female: Yes, I don't see how you can fill any of them in[inaudible].

Lecturer: Well, if you think that, then you'll see it's not a truth-functor. I mean, what's important is that the minute you get one question mark, it's not a truth-functor. So if you get more than one, it's even more obviously not a truth-functor. It's only if you're questioning whether it is a truth-functor that I'm interested in what you're saying.

I'm always interested in what you're saying.

Yes, I have put 'false's in there, but I see exactly what you're meaning, and you might prefer to put question marks in there. Either way, it doesn't matter. It's not a truth-functor.

(Slide 25) The sentence connective 'because' is not a truth-functor. It's not truth-functional. It's not a truth-functional operator. It's not a truth-functional connective. I'm using all these words interchangeably. Because in order to complete the truth table, we'd need to know the meaning of ‘P’ and ‘Q’, and we don't know the meaning of ‘P’ and ‘Q’, therefore we don't know. The truth values on their own are not sufficient to complete the truth table.

(Slide 26) Can you say which of these connectives is truth-functional, and give complete, partial, or blank truth tables as appropriate? I'm going to let you do that at home and you'll find the answers in your answer booklet, so you'll be able to do them without looking at the answers of course, and then check your answers against the answers in the booklet.

(Slide 27) There are five truth-functional sentence connectors – actually, there are more, but we're only going to do these five – in English. They are: 'it's not the case that', 'and', 'or’, 'if then', and 'if and only if'. These are the only five truth-functors. Let's have a look at another one, just to make sure that we know the meaning of the word 'not'.

So we've got 'not P', 'P'... we've only got two possible worlds here. It's either true or it's false. If ‘P’ is true, what's the truth value of 'not P'?

Male: False.

Female: False.

Lecturer: False. If ‘P’ is false, what's the truth value of...

Female: True.

Lecturer: It's true, isn't it? Do you see? So 'not' or 'it's not the case that' is another truth-functor.

'Or', 'if then', and 'if and only if', incidentally, we have to be careful with these other three. Sean said earlier – go on, say it again. I can see that you're dying to say it again.

Male: [Called 0:32:42] 'not P; Q'. You can still have ‘F’ and ‘T’, but then I could have 'not Q', i.e. 'not not-P', and I've gone from false back to true again, so two 'not's don't admit a third. It's a binary system.

Lecturer: Not in classical logic, no, but I've already said that in classical logic, we do assume 'bivalence', it's called – that there are only two truth values and there is no truth value gap. We assume that. That's just a...

Male: Well, is it not an axiom?

Lecturer: Yes. An axiom is something that you start with, you don't prove. A theorem is proved in a system. An axiom is taken for granted.

So you're seeing that you can fill up the truth table immediately for those two, no problem at all. But there is a problem for the others, and I can explain a bit of what I mean by looking at 'or'. Sean didn't say the thing I thought he was going to say.

Male: Oh.

Lecturer: It's all right. You said something else.

Male: It was that [inaudible]...

Lecturer: If we look at the world in which ‘P’ is true and ‘Q’ is true – no, let's look at this world first. So we're looking at world three, where ‘P’ is false and ‘Q’ is true. Is 'P or Q' true or false?

Male: True.

Female: True.

Male: True.

Lecturer: True. Okay. If we look at world two, where ‘P’ is true and ‘Q’ is false, what's the truth value of 'P or Q'?

Female: True.

Male: True.

Male: True.

Lecturer: True. If we look at world four, where ‘P’ is false and ‘Q’ is false, what's the truth value of 'P or Q'?

Female: False.

Male: False.

Lecturer: Good. It's looking good, isn't it? What about the world where ‘P’ is true and ‘Q’ is true? Is that true or false?

Male: It may be.

Male: False.

Lecturer: Do you know what you said earlier that I'm... okay, you think it's – the thing is, in English, 'or' is ambiguous, isn't it? It can be either inclusive or exclusive, which is what you said before, which is why I'm picking it up here.

So if I say to you, ‘You can have apple pie or ice cream’ and you say, ‘Oh, can't I have both?’ and I say, ‘Well, yes, if you like’, so there I'm not meaning it exclusively, am I? But if I say, ‘No, of course you can't’, I mean it exclusively.

In logic, we always say that it's inclusive. We always. So there's a stipulation there that you may disagree with, but that's the way it goes. There are good reasons for stipulating this. I'm not going to tell you what they are.

Male: Could you just do one of the others? Because I've come across inclusive and exclusive 'or's’, and I can't see how it would apply to the 'if then' or the [Cross talking]-

Lecturer: I’ll do the 'if and only if'.

Male: I can't quite see how that applies.

Lecturer: If we have 'P if and only if Q' and in the world where ‘P’ is true and ‘Q’ is true, what would be the truth value of...

Male: True. ‘If and only if Q’ is... that would be true.

Lecturer: Okay. If ‘P’ is true and ‘Q’ is false, what would be the truth value of 'P if and only if Q'?

Male: False.

Lecturer: False. If in that world... false.

Male: False.

Male: False.

Male: Yes.

Lecturer: And in this world?

Male: True.

Lecturer: Yes. You see? You can do it. I realise that not everyone was following that at the same time, but-

Male: But that's not how[inaudible]. Unlike the 'or', where you could argue about the two truths. I don't think you can argue about the two falses on the [Cross talking 0:36:48].

Lecturer: No, well, that's why I let you do it. It's fairly straightforward. I used the 'or' to show you a very simple ambiguity. If I tried to show you the 'if then', you would all be squealing, so I'm not going to do that. I mean, I'd pull authority and tell you what the truth table is for 'if then' but I won't do it right now. Let's move on.

So, what's important is that you feel you know the difference between a non-truth-functional sentence connective and a truth-functional sentence connective. That's the important thing for you to get here. You may feel lacking in confidence right now, but perfectly reasonable. I mean, you've had all of twenty minutes on it. But there's more in your handout and there's even more in the book! I have written that bit.

(Slide 28) So: propositional logic can only be used to formalise and evaluate deductive arguments, the validity of which depends solely on their sentential structure or propositional structure, and the connectives of which are all truth-functional. That summarises in one rather long sentence what you can use propositional logic for, and therefore what you can't use it for as well.

Okay? Any questions about that, or can I move on? Please let me move on! (Laughs). Good.

(Slide 29) Another reason it's important to set out an argument logic-book style is that it's very much easier to see whether propositional logic can be used to formalise it. So any argument can be analysed in the way you learned in the first session. Having analysed it, you might then see that it depends on its predicate structure, i.e., not only on its sentential structure or whatever. Much easier to see it once you've analysed it. Analysing an argument is a very useful thing to be able to do.

So the first thing we were going to learn in this session is to which arguments you can apply the propositional logic you're going to learn. (Slide 30) Second thing is to learn how to provide an interpretation by allocating sentence letters to the simple sentences constituting the relevant English arguments, thereby semi-formalising the sentence.

You need to know what an interpretation is, you need to know what sentence letters are, you need to know what simple sentences are, in order to know what a semi-formalisation is.

(Slide 31) To do this we've first got to identify the sentences that make up an argument. So, here we're looking at an argument. Quite a simple argument, but it's one that we can apply propositional logic to, because as we've seen already it depends entirely on its sentential structure, and all the sentence connectives used in it are truth-functional.

Okay. What are the sentences that make up this argument? Can anyone give me one of them?

Female: ‘It is Friday’.

Lecturer: 'It is Friday.' Okay. Any others?

Male: 'Marianne is wearing jeans'.

Lecturer: 'Marianne is wearing jeans.' Any others?

Male: No.

Lecturer: No? Okay. No other arguments. It's neither Friday nor am I wearing jeans, but that's a valid argument. Accept that?

Female: Kind of?

Lecturer: (Laughs). Okay. (Slide 32) So, these are the two sentences which constitute that argument. (Slide 33) To provide an interpretation, what we do is we provide a sentence letter for each of these arguments. So, that's a sentence letter, a capital letter from that part of the alphabet. Incidentally, it doesn't need to be that part of the alphabet. It's just, conventionally, we tend to use sentences from that part of the alphabet. You can use whatever sentences you like, but then I'll hate marking your work because if you're putting 'S's and 'P's and 'A's and 'B's where I'm using 'P's and 'Q's, it is really irritating.

Male: Is there any reason why that was chosen to be the convention, or was it just arbitrary?

Lecturer: I think it's just arbitrary. I mean, it isn't – there may have been a reason why it was chosen, but it's been chosen. It's not important. What is important is that the interpretation is by convention set out as, ‘Sentence letter – colon – space – sentence’. ‘Sentence letter – colon – space – sentence’.

Male: Is it always capital ‘P’?

Lecturer: Yes, always, and that's because when we have a predicate structure – so if you have, ‘Marianne is wearing jeans’, which of course you can't formalise except via one proposition, one sentence letter – but if I were doing predicate logic with you, that would be formalised, ‘Fa’. So the ‘a’ tends to be – so that means 'Marianne'. ‘a’ is ‘Marianne’, and ‘F’ is ‘is wearing jeans’. So the lower-case letter tends to be an individual constant. But you don't need to know that.

Male: We're not asking [inaudible] A is a function of F, therefore wouldn't the a be in brackets, no?

Lecturer: Some people would put it in brackets. I don't see that the brackets are necessary there.

Male: But I think [inaudible].

Lecturer: No. I can't-

Male: Not the same sort of function [inaudible]?

Lecturer: Okay, so that's an interpretation for that argument. So we can put 'interpretation' here. There are only two such sentences. You've got those. To make an interpretation, we give each sentence a sentence letter, and sentence letters stand for particular sentences, so actually there are also things called 'sentence variables', so that – Greek letters – tend to stand for any sentence. Whereas ‘P’ tends to stand for a particular sentence.

Again, you don't really need to know that, but if you're reading around what I'm saying and you see Greek letters being used instead of English letters, that's why.

(Slide 34) We use the interpretation to replace the sentences in the argument with sentence letters, so what we're doing is we're getting rid of the content of the argument. We know that deductive logic is content-neutral, or as I put it earlier, 'topic-neutral'. So actually, if we just get rid of the content it makes everything easier for us. We do that by replacing the sentences with sentence letters.

So let's do that. How would we do that here? What would the first premise be?

Male: 'If P then Q'.

Lecturer: 'If P then Q', because we replace every instance of, ‘It is Friday’ with ‘P’, and every instance of, ‘Marianne is wearing jeans’ with ‘Q’, don't we? So we get – what's the first premise?

Male: 'If P then Q'.

Female: 'If P then Q'.

Lecturer: 'If P then Q'. The second premise?

Male: [Cross talking].

Female: [Cross talking].

Lecturer: In conclusion...

Male: ‘Q’.

Male: ‘Q’.

Female: ‘Q’.

Lecturer: Yes. So, there's our argument. We've stripped it of its content and we can see the logical form of the argument so much more clearly having taken away the content of that argument.

(Slide 35) The only thing that's left that – so I said we would semi-formalise in doing that, of course we've still got these two English words in here so we haven't fully formalised the argument at the moment. We've also still got the labels, 'premise' and 'conclusion'. We need to formalise all that before we finally formalise the argument.

But what we've done here is we've semi-formalised it. We've replaced the content with sentence letters.

(Slide 36) I said earlier we'd try and semi-formalise the argument, the validity of which depends on its predicate structure. Do you remember the second argument? ‘Marianne is wearing jeans’. So what are the constituent sentences of this argument?

We're going back, we were looking at what the constituent sentences are of this argument.

Male: ‘Marianne is wearing jeans’?

Lecturer: ‘Marianne is wearing jeans’.

Male: ‘Marianne is the Director of Studies’...

Lecturer: ‘Marianne is D of S in P at OUDCE’. Okay. Anything else?

Male: [inaudible].

Lecturer: 'D-O-S in P at OUDCE is wearing jeans'. Okay, so the interpretation becomes...

We give each sentence letter, and the argument becomes...

Female: 'P and Q'.

Male: 'P and Q, therefore S'.

Lecturer: (Slide 37) P, Q, therefore S. The argument's just disappeared – whoops, 'therefore'. The argument's just disappeared, and of course it's disappeared because you're trying to formalise in propositional logic an argument that is not valid by virtue solely of its propositional form.

Do you remember, we identified this argument as valid by virtue of its predicate structure? In other words, we have to look *inside* the sentence in order to see that it's valid, and we just demonstrated that because if you try and do it in propositional logic the argument just disappears. It's not valid by virtue of its propositional structure; it's valid by virtue of something else, and therefore you can't use propositional logic to formalise it.

(Slide 38) Okay, here's a lovely argument. Let's semi-formalise this argument. Quite a big argument, but actually aren't you glad I've analysed it?

(Slide 39) Now, so what I've done is: I've identified – do you remember all the steps? I've identified the conclusion, then I've identified the premises. I've taken away the irrelevancies, so for example, ‘Unless you really thought she was such a perceptive cat that she'd understand that 'woof woof' means 'roll over'‘ is irrelevant, so we're getting rid of it. We don't want to waste our time formalising that when in fact it plays no role in the argument.

‘If you thought that then you're an idiot, but you're not an idiot, just twisted’ – I mean, again, that's 'P and not-P', in effect, 'not P and Q'. Anyway, believe me, this is the analysis of that argument. So, all the steps I gave you in the first session, I have applied to that argument and you might usefully, if you want, go back and look at the unanalysed argument, look at the analysed argument, look at the steps by which to analyse an argument, see how I got that from the original argument.

So you'll be glad that you don't have to analyse that argument, (Slide 40) but you do have to semi-formalise it now. So that's our task. I'm not sure whether to make you all do this yourselves or do it together. Shall we do it together?

Male: Yes.

Male: Yes.

Male: Yes.

Male: [inaudible].

Lecturer: (Slide 41) Okay, first we identify the constituent sentences. Let's do that. Let's go back to... I can't remember what I... okay, don't look at your sheets at the moment. Just look at this and tell me what the constituent sentences are.

Male: ‘She didn't want you to tickle her’.

Lecturer: Okay. ‘She didn't want you to tickle her’.

Male: ‘She didn't realise you were going to tickle her’.

Lecturer: ‘She didn't realise you were going to tickle her’.

Male: ‘You were going to tickle her the wrong way’.

Lecturer: Good. The first sentence in that, we've already got, haven't we? So, ‘You were going to tickle her the wrong way’ – ‘going to tickle her in the wrong way’.

Male: ‘You deserved to get scratched’.

Lecturer: And, ‘You deserved to get scratched’. Anything else?

Male: No.

Male: No.

Lecturer: No? Is that it? (Slide 42) Okay, so those are the constituent sentences. What do we do now?

Male: Pick [inaudible].

Lecturer: We provide an interpretation. (Slide 43) So we provide for each of those sentences a sentence letter, and we do it of course in accordance with the conventions. ‘P’ – colon – ‘Q’ – colon – ‘R’ – colon – ‘S’ – colon. Okay? That's an interpretation for that argument.

Male: There's also a 'not-P', isn't there?

Lecturer: Ah, but do we need another sentence letter for that?

Male: No, [Cross talking]-

Lecturer: No, we don't. We can use a truth-functional connective for that, so the fact that there's a ‘not-P’ and a ‘P’ in there is not something you need in the interpretation. That's something you use in the argument.

Okay. So, first we identify the constituent sentences, and that's, ‘She didn't realise you were going to tickle her’, ‘You were going to tickle her in the wrong way', 'You deserved to get scratched'. Well done, you see you got that all right.

Next, we provide the sentences with sentence letters, and again you've got that all right. Well done.

Now, we substitute the sentence letters for the sentences, thereby partly formalising the argument. Do you want to do that on your own, and then we'll do it together? So each of you do that on your own.

Somebody's just pointed out I did the same argument on the podcast, so you should all have finished by now. (Laughs).

(Slide 44) Okay, so there's the interpretation, and now we're substituting the sentence letters for the sentences, thereby partly formalising the argument, (Slide 45) and here is the argument with the sentences – sorry, the sentence letters are substituting for the sentences. (Slide 46) Is this what you've got?

Female: Yes.

Male: Yes.

Male: Yes.

Lecturer: Okay. Did anyone get something different from that?

Female: I've got that, but I don't understand it.

Lecturer: What don't you understand?

Female: I've got it, but I don't [Cross talking]...

Lecturer: Well, if you've got it then you must have understood it.

Female: [inaudible].

Female: [Cross talking]

Lecturer: What don't you understand, though?

Female: Well, why is it 'if not' in the conclusion? [Cross talking] going down.

Lecturer: ‘If not P, then S’, because ‘P’ is, ‘She *didn't* want you to tickle her’, and the conclusion is, ‘If she *did* want you to tickle her, then you deserved to get scratched’. Okay?

Female: So I got the form right, but I didn't quite understand what I was doing, obviously.

Lecturer: But do you understand now why-

Female: Oh, yes.

Lecturer: Why it's 'not-P' and ‘P'?

Female: Yes, I do. I'm a bit dim.

Male: [inaudible].

Male: Just ignore that [inaudible].

Lecturer: Okay.

Male: Is it the inclusive 'or' or the exclusive 'or' [inaudible]?

Lecturer: Oh, that's interesting. Okay.

Male: I couldn't understand why I got the conclusion, [inaudible] talking about exclusive 'or', do you know what I mean?

Lecturer: No. Hang on.

Male: Well, it's either ‘P’ or ‘Q’. It's both ‘P’ and ‘Q’.

Lecturer: Erm, no. It's an 'if then'.

Male: Oh, that's right.

Lecturer: It's not an 'or' – there's an 'or' in the premise one, but there's no 'or' in the – except actually, you can make an 'or' from 'if then' but we won't talk about that now.

If you've got anything else and you'd rather talk to me about it in the break, then that's what we can do.

Female: Mine's in red, by the way.

Lecturer: So you've done that. (Slide 47) Okay. So you've learned how to provide an interpretation and how to semi-formalise an argument. Well, that was quick, wasn't it? There's nothing difficult about those two things, but the third thing we've got to learn is to identify the correct symbols to represent the truth-functional connectives.

Okay, so at this stage you've formalised the content of the argument, but logical structure is still in English. (Slide 48) You've still got 'or's, 'and's, 'if then's, and 'not's in the argument, and what you need to learn is which symbols will formalise which sentence connectives.

(Slide 49) Here are the symbols for the truth-functional connectives that I gave you earlier. So, the negation, the truth-functional connective 'negation', or 'it is not the case that', has that symbol, and there are numbers of notational variants, incidentally. So if you go away and read around this, you may see other symbols. Again, I'm not sure if I give the – I give the notational variants in the handouts, I think, just so that if you're reading a book that uses different symbols you can see which symbol it is. But that's the one we're going to use and that's because my computer makes it easier to find that one than any other.

This is the conjunction truth-functional connective which is 'and' in English. It's that symbol.

The 'or' is the disjunction sentence connective, and we use the vel.

The conditional 'if then' is an arrow, like that.

The biconditional 'if and only if', we use a double-ended arrow there.

So those are the symbols that we use, but there are notational variants for each one of them, which won't bother us now but if you do any reading around this you may meet one of these, and it won't mean anything to you until you check that it is indeed a notational variant.

Male: In philosophy, is the single arrow 'implies that'?

Lecturer: Yes.

(Slide 50) So, which truth-functor symbols would you use to formalise these truth-functional connectives? Is that a conjunction, disjunction, negation, conditional, or bi-conditional?

Male: ‘P v Q’.

Female: Disjunction.

Lecturer: It's a disjunction.

Is that a conditional, a conjunction, a disjunction...

Male: Conditional.

Female: Conditional.

Lecturer: That's a conditional. It's an arrow. That's right.

‘If R then S’?

Male: Conditional.

Female: Conditional.

Lecturer: It's another conditional, isn't it?

‘If not P then S’?

Male: Negation. [inaudible] conditional.

Female: Negation.

Lecturer: That's a negation and that's a conditional. That's right.(Slide 51) So these are the symbols that we're using. So, there's a disjunction, a conditional, a conditional, and another conditional, and that has a negated antecedent.

Okay, so that's fairly straightforward-

Male: Can I just check: do the brackets have any real significance?

Lecturer: (Slide 52) Yes, the brackets have a huge significance. Very important to the – you'll see that I put brackets in for every binary ‘truth-functor. Are you getting the language? That's a binary truth-functor because it takes two sentence letters to make a sentence. I use a pair of brackets.

So here we have another binary truth-functor and again I've used a pair of brackets. Another binary truth-functor, another pair of brackets.

Here, there's a unary truth-functor for which I don't need brackets, and a binary truth-functor for which I do use brackets. But the last formula there is ambiguous without the brackets. If I put that without brackets, it could be that or it could be that.

So the scope of the negation operator could be just ‘P’, or it could be the whole conditional. ‘P – arrow – S’.

Male: [inaudible] the negation of the affirmation [inaudible], no matter which way you write it, isn't it? The negation of the conjunction is the conjunction of the negations.

Lecturer: Well, but we haven't got a conjunction here. What we've got is a conditional.

Male: Oh, sorry. Yes.

Lecturer: So we've got, ‘Not P – arrow – S’, but that is ambiguous. It could be either of these formulae, and until we put the brackets in we don't know which one it is. So when we look back at the argument itself, ‘If not P then S’, we see that the scope of the negation operator is just the ‘P’. Otherwise it would be, ‘Not if P then S’.

Male: Presumably, the brackets are very important if you had, ‘P or Q and R or S’.

Lecturer: Yes. Whenever you get more than one operator, you probably – brackets are very, very important. I've always made students put brackets in the minute they get a binary truth-functor just because it's good practice.

Male: Should you work [-inaudible] read inside the working side, sorry, the brackets first, and then go on to the rest of the expression?

Lecturer: We'll talk about that later. You're quite right to think that's very important.

Okay, so – I've gone back, haven't I? The brackets are very important because we can't use commas and things like that to disambiguate things in the language of the propositional logic. We've got to use brackets.

(Slide 53) But what I haven't told you is all the pitfalls that you're going to fall into in interpreting English arguments in the language of the propositional calculus. There are so many of them, which is why this is the hardest thing you'll ever learn to do.

Once you can do this, evaluating the arguments, believe me, is very, very easy. Doing this is difficult because trying to put a natural language – English – into a logically perfect language, because propositional logic is a perfect language, there's no ambiguity, every truth-functor has a very specific truth table definition. It's what Russell called a 'logically perfect language', and trying to squeeze a natural language into a logically perfect language is very difficult.

What's more, you will never, ever get it right. In many cases, there is no such thing as 'right'. But there are pitfalls that we can see immediately, and you just have to learn these. I've given you, on your handouts, about four pages of why it's difficult to do the 'and'.

But here are a few highlights. 'Not', in English, is ambiguous. If I say, ‘It's not the case the King of France is bald’, that means one thing. If I say, ‘The King of France is not bald’, do you see the 'not' has small scope, small scope in the latter case? So we're saying of the King of France that he's *not bald*, whereas if we're saying, ‘It's not the case that the King of France is bald’ that's consistent with there being no King of France, so we're cancelling all the implications.

So 'not' in English can either be a contradictory, where you can't have 'P or not-P' or it can be a contrary, so if I say, ‘You ought to kiss her and it's not the case you ought to kiss her’, that's not a contradictory because both those things might be true. You ought to kiss her because she's your sister; you ought not to kiss her because she's been a very naughty girl. Do you see that both the – in a contradictory, they can't both be true together and they can't both be false together.

So we've got to use 'it's not the case' instead of just 'not', and that's why the truth-functor symbol for 'not' is always understood in English to be 'it's not the case that' rather than 'not'.

That's the first pitfall. So you can only really – we ignore this all the time, incidentally – you're only supposed to translate contradictories with 'not', not contraries.

'And', on the other hand, connects things other than sentences. So, ‘the cat is black and white’ – the black and white there is linked by 'and' but, neither ‘black’ nor ‘white’ is a sentence. 'Black' and 'white' are both predicates. So 'and' can be a predicate connective as well as a sentence connective, and you don't use the logical conjunction sign for any 'and' that is a predicate connective as opposed to a sentence connective. You see what I mean?

Also, if I say – well, let me write it so you can see it – ‘Philosophers who drink are logical’. ‘Philosophers, who drink, are logical’. Do you see I've put commas in that one? One of those sentences can be translated with the conjunction sign, and the other can't. Which is which?

Male: The first.

Lecturer: Let me read them again. ‘Philosophers who drink are logical’. ‘Philosophers, who drink, are logical’.

Male: The second.

Male: The second.

Male: It's a [inaudible] conjunction.

Lecturer: The second is the conjunction.

Male: The philosophers, who drink, and the philosophers, are logical. They're two [Cross talking].

Lecturer: That's right, and that's because here, you're saying of philosophers who drink, that they're logical, whereas here you're saying two things about philosophers: one is that they drink, and the other is that they're logical. See what I mean?

So that one is two sentences: ‘Philosophers drink’ and, ‘Philosophers are logical’. Whereas that is one sentence: ‘Philosophers who drink are logical’. Are you with me? Do you see how 'and' can be used for that one but not that one?

Okay, 'or' – we've already looked at that one. We will use a vel for an exclusive 'or', but we really shouldn't, and we're missing the implication that it is possibly exclusive. So we quite often ignore implications, so things that we would say in English, we can't say in the language of propositional logic.

If I say, ‘If I drop this glass onto concrete’ – just to forestall objections – ‘If I drop this glass onto concrete, it'll break’, I can't use the arrow of the conditional for that, and the reason for that is that the two sentences are not independent. Do you see that, ‘It will break only if I drop it’, so the condition of its breaking is dependent upon my dropping it. It's not, ‘It will break’. It's not an independent sentence. In order to use a sentence connective, the two sentences have to be independent.

Last pitfall I'll mention, but again it's in your handouts – it's even more in the book, it's even more in the podcast – you will be able to find these things out. We don't have time to do them now. People got so sick when I used to say to undergraduates all the time: whenever they saw an 'only', whenever they saw an 'if', they would reach for the biconditional, the two-sided arrow.

But the biconditional should only be used if you've got a necessary and a sufficient condition. In other words, if the arrow goes in both directions. ‘If P, then Q’ and, ‘If Q, then P’. ‘P if and only if Q’. That is such a common error. You'll find yourself doing it, I'm sure, because it just seems to be a psychological fact about human beings, that they make that confusion until they know what they're doing.

So, lots of ways you can find out more about this. We don't have time to do it now, but translating English arguments into the propositional language is not easy. It's the hardest thing you'll ever do, and the reason it's hard is because it's not systematic in the way that everything else is.

(Slide 54) The final thing we need to learn in this session – you've learned two of the things – the final thing that we need to learn is how to represent English arguments as a sequent of propositional logic.

A sequent is an argument claim. So our fully formalised argument looks like this, (Slide 55) but I've put 'fully' in scare quotes because it's actually not fully formalised, because English is what tells you which are the premises and which are the conclusions, isn't it? They're labelled in English so that you know what the relation is between the sentences of the argument.

What we want to do now is to formalise the relation between premises and conclusion. We do that like this. Lo! She waves her magic wand (Slide 56)

This symbol here is called a semantic turnstile, and it's the symbol for – well, it's not actually. There are all sorts of pitfalls for interpreting it as 'therefore', but I have learned that people understand it very easily if I say, ‘that means 'therefore'.’

(Slide 57) It doesn't, actually; this means there is no logically possible situation in which the formulae on the left-hand side of the sequent – that's (The turnstile) the sequent – are all true, and the formulae on the right-hand of the sequent false.

This means there is no logically possible situation in which all of these formulae, the one on the left-hand side of the sequent, are all true, and this formula is false. You should be getting a feel for what that means now, because don't forget: what we want to know is whether the argument with which we started is valid or not. We know that it will be valid if there's no possible situation in which the premises are true and the conclusion false.

We've formalised the premises and the conclusion, and we're now representing the relation between the premise and conclusion in this sequent. So this is the argument claim.

The claim is: there's no possible situation in which these are true and that is false. That's the claim that we now need to test. Are you with me?

Male: Could I be slightly provocative? If you use 'therefore' for that [inaudible], could you replace the commas with 'and's?

Lecturer: Certainly not! No, no, no. No, because 'and' is English.

Male: Yes, but I'm saying: if you've got a [inaudible], you could have a single sentence, so, ‘P or Q and [Cross talking], if Q then R and if R then S’, then the turnstile?

Lecturer: Yes, you can do that, but you'd have to add another two pairs of brackets.

Male: Sorry, I was going backwards on that one.

Lecturer: Yes, I'd really rather you didn't do that. But, yes, you can do it [Cross talking].

Male: If you were translating back into English, [Cross talking]-

Lecturer: But really, the truth table you'd end up with [inaudible] then is just so horrible you just don't want to think about it, and you'll know what I mean by that in a minute.

Do you understand why that is the argument claim? We've formalised the argument claim as well as the content of the argument. Actually, we haven't fully formalised it, because we're still talking about truth and falsehood. Well, can you tell me what truth is?

Male: [inaudible].

Female: Valid.

Lecturer: I don't think – no, I don't think truth is one. I think truth is something that – actually, it's very difficult to say what truth is, and it's very difficult to say what falsehood is. So let's get rid of truth and the notions of truth and falsehood, and let's say, ‘Here's another argument’. Oh, okay. That's the semantic sequent. I don't know why I've got this slide here, but anyway, there it is. Okay, that's the semantic sequent, and here's a syntactic sequent. (Slide 58)

You'll see that the only difference is that here I've got a *syntactic* turnstile. There's only one crossbar on it. So there's the semantic sequent – 'semantic', to do with meaning and truth – and here's the syntactic sequent – 'syntactic' is to do with grammar and structure.

What we've done here is to eliminate the notions of truth and falsehood and that's because of the way this is understood. (Slide 59) The way this is understood, it's still an argument claim, the same argument claim, but that's got to be read as, ‘The set consisting of these formulae is closed’.

Note that what I've done here is I've taken out the syntactic sequent and I've negated the conclusion, and I'll explain all that later on. But the full formalisation of the argument is that, because I can tell you how to evaluate that without using any English at all. I don't have to use the words 'truth' and 'falsehood', I don't have to use anything but the rules of the propositional calculus. Whereas on that sequent, the semantic sequent, I still have to use the notions of truth, falsehood, possibility, and English notions that we don't really understand.

Male: So can you always replace that sequent by the other one?

Lecturer: I think so, yes. I'm at the stage where it is possible that what I'm saying is false, but I'm not sure. I think so. If that is proven to be correct, then its equivalent syntactic sequent will always be correct as well.

Male: So it really just boils down to your claim about what truth is.

Lecturer: Yes. I mean, what you're doing here, you're still talking about truth because that's defined as, ‘there is no logically possible structure in which those are true and that's false’, okay, so the truth is still in the definition. Whereas once you've got this, then you're saying, ‘the set consisting of that lot and the negation of that is closed’. I'll show you what is meant by that later on.

Male: Closing [inaudible]?

Lecturer: With the idea of syntactic closure. Syntactic con- well, if I say 'contradiction' I've brought English back. Closure, I'll explain later on.

Male: Can you go back to [1inaudible] you've got there, ‘The semantic turnstile is understood as...’ and you've given a sentence to explain what that means. Is there an equivalent sentence for the syntactic sentence [inaudible], and is that it?

Lecturer: That's it. It's not got much English in it, has it? I mean, the only English in it is the English that I need to define it for you.

Male: But you can remove that, couldn't you? [Cross talking]

Lecturer: I will remove it. I'll remove it very soon.

Male: The syntactic the adjective and syntax thing [inaudible], which is [Cross talking]-

Lecturer: Yes. Grammar or structure as opposed to semantic, which is truth or meaning.

(Slide 60) Okay, so we started with that, then we ended with that, and you now know – roughly – how to get from that to that. You'll be able to do it yourself when you go home tonight and you practice on those two arguments with which we started this session.

Can you see how we can go from that to that, and that that hasn't captured the meaning totally of that because we've left out a lot of irrelevancies here? I've said that in using the truth-functor symbols we're occasionally not capturing implications of English.

But if you want to know whether that's a valid argument, it's much better to test that than it is to have a go at trying to say whether that's valid. Because after the break, I'm going to give you rules by which to test that, which should be completely foolproof.

(Slide 61) Okay, so, you know how to distinguish arguments that can be formalised in the language of propositional logic from those that can't. You know how to provide an interpretation by allocating sentence letters to the sentences constituting the English arguments. You know how to identify the correct symbols to represent the truth-functional connectives, from which the complex sentences of the relevant English arguments are constituted. Then you know how to represent the English argument as an argument claim, as a sequent of propositional logic – either a syntactic sequent or a semantic sequent.

(Slide 62) Huh. As I've said, mine's the red.

(Slide 63) There's an argument to practice on at home, and I've given the answer in the answer booklet, which I haven't given out yet. They're just on the table. Good.

Male: Thank you.

Female: Thank you.